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A TREATISE
ON THE
COMPOUND STEAM ENGINE,
IN TWO PARTS.

PART I.

By JOHN TURNBULL, Jr.,
WITH
METHOD OF FINDING THE RELATIVE CYLINDER VOLUMES.
REVISED, WITH ADDITIONS.

PART II.

GRAPHICAL AND ANALYTICAL TREATMENT.

ILLUSTRATED WITH DIAGRAMS, TABLES, ETC.
Including Rotative and Pumping Non-Rotative Engines.
By S. W. ROBINSON, C. E.,
Prof. Mech. Eng., Ohio State University.

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PREFACE.

This little volume treats only of the Woolf compound engine, that is, one in which the pistons move simultaneously, except in Cases V., VI. and VIII., for pumping engines; and with the exception of V. and VI. the engines may be rotative; that is, have a main shaft, fly-wheel, etc. The rotative engines may serve for pumping engines, or any other purpose.

No case is treated in which the cranks may be otherwise than all on their lines of centers simultaneously except case VIII.

The treatment has been by analysis, so far as excessively complex formulas could be avoided. When expansion is introduced in the low-pressure cylinder the formulas involve too many terms, and to account for cushion and clearance in addition, the analytical formulas become useless for practice, if not impossible to obtain.

For this reason the graphical method has been resorted to in treating of the details respecting the effects of cushion, clearance and expansion in both cylinders, together with an intermediate receiver.

The graphical method is believed to be capable of great service in discussing the effects of matters that cannot be conveniently reached by analysis. For this reason pains have been taken to extend the graphical diagrams to include considerable detail. It is believed that with the diagrams herein presented, the most fastidious investigator or designer can correctly lay out the theoretical diagrams for any engine of the type here discussed; and settle questions of relative efficiency of various modifications as to design or dimensions of parts thermodynamically involved. In Part I. a method and tables are given for finding the relative volumes of cylinders for equally dividing the work between the two cylinders on the supposition of no receiver, clearance, cushion, superheating, reheating, fluid friction, or back pressure of exhaust. This will serve as a guide to an approximate result for a practical case where all the above-named qualifying causes cannot be entirely avoided, With the first approximations thus obtained, Part II. will serve for securing a more extended refinement of approximation.

S. W. R.

PART I.

A

TREATISE ON THE COMPOUND STEAM ENGINE,

BY

JOHN TURNBULL, Jr.,

WITH

METHOD OF FINDING THE RELATIVE CYLINDER VOLUMES.

REVISED, WITH ADDITIONS.

THE WOOLF COMPOUND ENGINE WITHOUT RECEIVER.

THE compound engine—whatever diversity of opinion may be held by engineers and others as to its merits as an economical expansive engine—has attracted towards itself a very considerable share of attention, from the superior results that have been obtained by it in many instances; and it is reasonable to suppose that, when a certain degree of perfection has once been attained in the manufacture of any machine, or economy secured by any new arrangement of its parts, similar machines can be so constructed as to give out the same results, if proper care is taken that the same arrangement and construction is faithfully carried out as in that of the more perfect machine. And when that degree of economy has not been obtained from a compound engine which had reasonably been expected, it would, no doubt, be found, if proper inquiry were made, that the fault lay, not in the principle that had been adopted, but that sufficient skill had not been exercised in properly proportioning the different parts through which the steam had to pass or come in contact on its way from the boiler to the condenser, and that sufficient means had not been employed to prevent or replace any waste of heat from condensation and other causes.

As the compound engine is being now so universally adopted in the Mercantile Marine Service, and a knowledge of its principles absolutely necessary by those engaged in attending it, we will, in the following remarks, explain these principles in as simple a manner as possible, and institute a comparison between the respective merits of the single-cylinder expansive condensing engine and the compound engine.

The compound engine is a high and low-pressure condensing engine, having two ordinary steam cylinders, the smaller or high-pressure cylinder communicating direct with the boiler, the larger or low-pressure cylinder direct with the condenser, and both with each other. The steam is admitted freely from the boiler into the high-pressure cylinder until the piston has been moved through a certain distance where the valve is so regulated that the communication with the boiler is entirely shut off, and the remainder of the space to be passed through by the piston is performed by the expansion of the steam now shut up in the cylinder, and which, after doing its work in this cylinder, passes on to the low-pressure cylinder, where it does an equal or proportionate quantity of work, and then passes into the condenser. This is the type known as the Woolf compound engine.

It has been found from modern practice that when the length of stroke of both cylinders is the same, it is necessary that the condensing cylinder be about three times greater in area than the high-pressure one, and this proportion is best suited when the steam employed is from 45 to 50 lbs. pressure above the atmosphere, and cutting off the steam after being admitted during $\frac{1}{8}$ of the stroke in the high-pressure cylinder. When the steam to be employed is of a less pressure, but the point of cut-off the same, then the relative proportions of the cylinders must be nearer to each other, and the reverse when steam of a greater pressure is to be used.

THE SINGLE CYLINDER EXPANSIVE ENGINE.

To get the maximum of economy out of any class of expansive condensing engine, the pressure of steam and point of cut-off must be so regulated that the steam passes into the condenser at the end of the stroke at a pressure not exceeding 5 lbs. above a perfect vacuum, and with steam at 45 lbs. pressure above the atmosphere, which is equal to 60 lbs. pressure above a perfect vacuum (the pressure of the atmosphere being considered as equal to 15 lbs. on the square inch), and a terminal pressure of 5 lbs., we get, according to Mariotte's law, 12 expansions, because the pressure at the end of the stroke is 12 times lass than what it was at the point of cut-off, and is expressed by the formula

$$\frac{P_1}{P_2} = R.$$

where P_1=pressure at point of cut-off,

P_2=terminal pressure, and R=ratio or whole number of expansions. To simplify this inquiry the law of Mariotte has been assumed throughout, viz., that the product of simultaneous pressures and volumes is constant. Hence as the pressure of dry saturated steam varies nearly inversely as the space it occupies, the steam will now fill nearly 12 times the space it originally occupied at a pressure equal to $\frac{1}{12}$th of the original pressure.

On reference to the annexed table of average or mean pressures, it will be seen that steam admitted at 60 lbs. pressure, and cut-off at $\frac{1}{12}$th part of the stroke exerts an average pressure=17.32 lbs. per sq. in. on the piston throughout the whole stroke, and although this is about 3½ times less work than would have been done had the steam been used at the full pressure of 60 lbs. throughout the whole length of the stroke, still only a 12th part of the cylinder's contents had been filled from the boiler, and the power required is thus got by working the steam expansively, at a saving equal to about 3½ to 1. (See Table A.)

TABLE A.—AVERAGE OR MEAN PRESSURE UPON PISTON DURING ONE STROKE.*

For any Rate of Expansion of Air at Unvarying Temperature. For Approximate Values for Steam Remaining Dry and Saturated During Expansion, Reduce the Values Given in the Table by 4 per cent. Thus the "29" at the foot under "100" would become $27\frac{7}{8}$.

Steam cut off at	Absolute Pressure in Lbs. at Commencement of Stroke.														
	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
$\frac{1}{3}$	21	$24\frac{1}{2}$	28	$31\frac{1}{2}$	35	$38\frac{1}{2}$	42	$45\frac{1}{2}$	49	$52\frac{1}{2}$	56	$59\frac{1}{2}$	63	$66\frac{1}{2}$	70
$\frac{2}{3}$	$28\frac{1}{4}$	$32\frac{3}{4}$	$37\frac{1}{2}$	42	$46\frac{3}{4}$	$51\frac{1}{2}$	$56\frac{1}{4}$	61	$65\frac{1}{2}$	$70\frac{1}{4}$	75	$79\frac{1}{2}$	$84\frac{1}{2}$	89	$93\frac{3}{4}$
$\frac{1}{4}$	$17\frac{3}{4}$	$20\frac{3}{4}$	$23\frac{3}{4}$	$26\frac{3}{4}$	$29\frac{3}{4}$	$32\frac{3}{4}$	$35\frac{3}{4}$	$38\frac{3}{4}$	$41\frac{3}{4}$	$44\frac{3}{4}$	$47\frac{3}{4}$	$50\frac{3}{4}$	$53\frac{3}{4}$	$56\frac{3}{4}$	$59\frac{3}{4}$
$\frac{1}{2}$	$25\frac{1}{4}$	$29\frac{1}{2}$	$33\frac{3}{4}$	38	$42\frac{1}{4}$	$46\frac{1}{2}$	$50\frac{3}{4}$	55	$59\frac{1}{4}$	$63\frac{1}{2}$	$67\frac{3}{4}$	72	$76\frac{1}{4}$	$80\frac{1}{2}$	$84\frac{3}{4}$
$\frac{3}{4}$	$28\frac{3}{4}$	$33\frac{1}{2}$	$38\frac{1}{2}$	$43\frac{1}{4}$	$48\frac{1}{4}$	53	$57\frac{3}{4}$	$62\frac{1}{2}$	$67\frac{1}{2}$	$72\frac{1}{4}$	$77\frac{1}{4}$	82	87	$91\frac{3}{4}$	$96\frac{1}{2}$
$\frac{1}{5}$	$15\frac{1}{2}$	$18\frac{1}{4}$	$20\frac{3}{4}$	$23\frac{1}{2}$	26	$28\frac{1}{2}$	$31\frac{1}{4}$	34	$36\frac{1}{2}$	39	$41\frac{3}{4}$	$44\frac{1}{4}$	47	$49\frac{1}{2}$	$52\frac{1}{4}$
$\frac{2}{5}$	23	$26\frac{3}{4}$	$30\frac{1}{2}$	$34\frac{1}{2}$	$38\frac{1}{4}$	42	46	$49\frac{3}{4}$	$53\frac{1}{2}$	$57\frac{1}{2}$	$61\frac{1}{4}$	65	69	$72\frac{3}{4}$	$76\frac{1}{2}$
$\frac{3}{5}$	27	$31\frac{1}{2}$	$36\frac{1}{4}$	$40\frac{3}{4}$	$45\frac{1}{4}$	$49\frac{3}{4}$	$54\frac{1}{4}$	$58\frac{3}{4}$	$63\frac{1}{4}$	$67\frac{3}{4}$	$72\frac{1}{2}$	77	$81\frac{1}{2}$	86	$90\frac{1}{2}$
$\frac{4}{5}$	$29\frac{1}{4}$	$34\frac{1}{4}$	39	44	49	$53\frac{3}{4}$	$58\frac{1}{2}$	$63\frac{1}{2}$	$68\frac{1}{2}$	$73\frac{3}{4}$	$78\frac{1}{4}$	83	88	$92\frac{3}{4}$	$97\frac{3}{4}$
$\frac{1}{6}$	14	$16\frac{1}{4}$	$18\frac{1}{2}$	$20\frac{3}{4}$	$23\frac{1}{4}$	$25\frac{1}{2}$	$27\frac{3}{4}$	$30\frac{1}{4}$	$32\frac{1}{2}$	$34\frac{3}{4}$	$37\frac{1}{4}$	$39\frac{1}{2}$	$41\frac{3}{4}$	$44\frac{1}{4}$	$46\frac{1}{2}$
$\frac{5}{6}$	$29\frac{1}{2}$	$34\frac{1}{2}$	$39\frac{1}{2}$	$44\frac{1}{4}$	$49\frac{1}{4}$	54	59	64	69	$73\frac{3}{4}$	$78\frac{3}{4}$	$83\frac{3}{4}$	$88\frac{3}{4}$	$93\frac{1}{2}$	$98\frac{1}{2}$
$\frac{1}{7}$	$12\frac{1}{2}$	$14\frac{3}{4}$	$16\frac{3}{4}$	$18\frac{3}{4}$	21	$23\frac{1}{4}$	$25\frac{1}{4}$	$27\frac{1}{4}$	$29\frac{1}{2}$	$31\frac{1}{2}$	$33\frac{1}{2}$	$35\frac{3}{4}$	$37\frac{3}{4}$	40	42
$\frac{2}{7}$	$19\frac{1}{4}$	$22\frac{1}{2}$	$25\frac{3}{4}$	$28\frac{3}{4}$	32	$35\frac{1}{4}$	$38\frac{1}{2}$	$41\frac{3}{4}$	45	$48\frac{1}{4}$	$51\frac{1}{2}$	$54\frac{1}{2}$	$57\frac{3}{4}$	61	$64\frac{1}{4}$
$\frac{3}{7}$	$23\frac{3}{4}$	$27\frac{3}{4}$	$31\frac{1}{2}$	$35\frac{1}{2}$	$39\frac{1}{2}$	$43\frac{1}{2}$	$47\frac{1}{2}$	$51\frac{1}{2}$	$55\frac{1}{4}$	$59\frac{1}{4}$	$63\frac{1}{4}$	$67\frac{1}{4}$	$71\frac{1}{4}$	$75\frac{1}{4}$	79

upon the Piston.

4/7	26¾	31¼	35½	40	44½	49	53½	57¾	62½	66¾	71¼	75¾	80	84½	89	Average Absolute Pressure in Lbs.
5/7	28½	33½	38¼	42¾	47¾	52½	57¼	62	66¾	71½	76¼	81	85¾	90¾	95½	
6/7	29½	34½	39½	44½	49½	54¼	59¼	63¾	69¼	74¼	79	84	89	93¾	98¾	
1/8	11½	13½	15¼	17¼	19¼	21¼	23	25	27	28¾	30¾	32¾	34½	36½	38½	
3/8	22¼	26	29¾	33½	37	40¾	44½	48¼	52	55¾	59½	63	66¾	70½	74¼	
5/8	27½	32	36¾	41¼	45½	50½	55¼	59¾	64¼	68¾	73½	78	82½	87¼	91¾	
7/8	29¾	34¾	39½	44½	49½	54½	59½	64½	69¼	74¼	79¼	84¼	89¼	94¼	99	
1/9	10½	12¼	14¼	15¾	17¾	19½	21¼	23	24¾	26½	28¼	30¼	31¾	33¾	35½	
2/9	16½	19¼	22¼	25	27¾	30½	33¼	36	38¾	41¾	44½	47¼	50	52¾	55½	
4/9	24	28	32	36	40¼	44¼	48¼	52¼	56¼	60¼	64¼	68¼	72¼	76¼	80½	
5/9	26¼	30¾	35¼	39½	44	48½	52¾	56½	61¾	66	70½	74¾	79¼	83¾	88	
7/9	29	34	38¾	43¾	48½	53½	58¼	63¼	68	72¾	77¾	82¾	87½	92¼	97¼	
8/9	29¾	34¾	39¾	44½	49½	54½	59½	64½	69½	74½	79¼	84¼	89¼	94¼	99¼	
1/11	9¼	10¾	12¼	13¾	15¼	16¾	18½	20	21½	23	24½	26¼	27¾	29¼	30¾	
2/11	14¾	17¼	19½	22	24½	27	29½	31¾	34¼	36¾	39¼	41¾	44¼	46¾	49	
3/11	18¾	21¾	25	28	31¼	34¼	37½	40¾	43¾	47	50	53¼	56¼	59½	62½	
4/11	21¾	25½	29¼	32¾	36½	40¼	43¾	47½	51	54¾	58½	62	65¾	69½	73	
5/11	24¼	28¼	32½	36½	40½	44½	48¾	52¾	56¾	60¾	65	69	73	77	81¼	
6/11	26¼	30½	35	39¼	43¾	48	52½	56¾	61¼	65½	70	74¼	78¾	83	87½	
7/11	27½	32¼	36¾	41½	46	50¾	55¼	60	64½	69¼	73¾	78½	83	87¾	92¼	
8/11	28¾	33½	38¼	43	47¾	52¾	57½	62¼	67	71¾	76¾	81½	86¼	91	95¾	
9/11	29¼	34¼	39¼	44	49	54	58¾	63¾	68¾	73½	78½	83½	88¼	93¼	98¼	
1/12	8½	10	11½	13	14½	15¾	17¼	18¾	20¼	21¾	23¼	24½	26	27½	29	

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* This includes the action in both cylinders if compound, the same as though there were but one cylinder and no dead space.

TABLE A—*Continued.*

Steam cut off at	Absolute Pressure in Lbs. at Commencement of Stroke.				
	110	120	130	140	150
1/3	77½	84	91	98	105
2/3	103	111½	121¾	131	140½
1/4	65½	71½	77½	83½	89½
1/2	93¼	101½	110	118½	127
3/4	106¼	115¾	125½	135¼	144¾
1/5	57¼	62½	67¾	73	78½
2/5	84¼	91¾	99½	107¼	115
3/5	99½	108¾	117¾	126¾	135
4/5	107½	117½	127¼	136¾	146¾
1/6	51¼	55¾	60½	65	69¾
5/6	108¼	118¼	128	137¾	147
1/7	46¼	50½	54¾	58¾	63
2/7	70¾	77¼	83½	90	96½
3/7	87	94¼	102¼	110¾	118¾
4/7	98	106¾	115¾	124¾	133½
5/7	105	114¼	124	133½	143¼
6/7	108¾	118¾	128½	138½	148¼
1/8	42¾	46¼	50	54½	57¾
3/8	81¾	89	96½	104	111½
5/8	101	110¼	119½	128½	137¾
7/8	109	119	128¾	138¾	148¾
1/9	39	42½	46	49½	53¼
2/9	61	66¾	72¼	77¾	83
4/9	88½	96½	104½	112½	120½
5/9	97	105¾	114½	123¼	132¼
7/9	107	116¾	126½	136¼	145
8/9	109¼	119¼	129	139	149
1/11	33¾	37	40	43¼	46¼
2/11	54	59	63¾	68¾	73¾
3/11	68¾	75¼	81½	87¾	94
4/11	80¼	87¾	95	102¼	109½
5/11	89¼	97½	105½	113¾	121¾
6/11	96¼	105	113¾	122½	131
7/11	101½	110¾	120	129¼	138½
8/11	105¼	115	124½	134¼	143½
9/11	108	117¾	127½	137½	147¼
1/12	31¾	34¾	37¾	40½	43½

Average Absolute Pressure in Lbs. upon the Piston.

TO CALCULATE THE MEAN PRESSURE FOR ONE STROKE OF THE SINGLE CYLINDER ENGINE.

As the point of cut-off may be different from any of those shown in the table, it is as well that the student should be in possession of a simple formula for ascertaining the average pressure for himself at any time, and the following is given to find out the mean pressure during a stroke in lbs. per sq. in. Let

L = Whole length of stroke in inches.

l = Distance traveled by piston before the steam is cut off, in inches.

R = Ratio or number of expansions $= \frac{L}{l}$.

H = Hyperbolic lorgarithm of R.

P_1 = Initial pressure of steam in lbs. per sq. in.

p_m = Mean pressure for the stroke in lbs. per sq. in.

Then, $p_m = P_1 \frac{1 + H^*}{R}$.

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* This formula is based upon the law of Mariotte, and the Woolf type of compound engine.

A table of hyperbolic logarithms is also annexed so that H may be found without any difficulty.

TABLE B.—OF HYPERBOLIC LOGARITHMS.

The Hyperbolic Logarithm of a number is found by multiplying the common Logarithm of the number by 2.30259.

No.	Logarithm.	No.	Logarithm.
1.1	.0953102	3.8	1.3350010
1.2	.1823215	3.9	1.3609765
1.3	.2623642	4.0	1.3862943
1.4	.3364722	4.1	1.4109869
1.5	.4054652	4.2	1.4350845
1.6	.4700036	4.3	1.4586149
1.7	.5306282	4.4	1.4816045
1.8	.5877866	4.5	1.5040773
1.9	.6418538	4.6	1.5260563
2.0	.6931472	4.7	1.5475625
2.1	.7419373	4.8	1.5686159
2.2	.7884573	4.9	1.5892352
2.3	.8329090	5.0	1.6094379
2.4	.8754686	5.1	1.6292405
2.5	.9162907	5.2	1.6486586
2.6	.9555113	5.3	1.6677068
2.7	.9932518	5.4	1.6863989
2.8	1.0296913	5.5	1.7047481
2.9	1.0647107	5.6	1.7227665
3.0	1.0986124	5.7	1.7404661
3.1	1.1314021	5.8	1.7578579
3.2	1.1631508	5.9	1.7749523
3.3	1.1939224	6.0	1.7917595
3.4	1.2237754	6.1	1.8082887
3.5	1.2527629	6.2	1.8245493
3.6	1.2809338	6.3	1.8405496
3.7	1.3083328	6.4	1.8562979

No.	Logarithm.	No.	Logarithm.
6.5	1.8718021	9.3	2.2300144
6.6	1.8870697	9.4	2.2407096
6.7	1.9021075	9.5	2.2512907
6.8	1.9169226	9.6	2.2617631
6.9	1.9315214	9.7	2.2721258
7.0	1.9459100	9.8	2.2823823
7.1	1.9600947	9.9	2.2925347
7.2	1.9740810	10.0	2.3025851
7.3	1.9878743	11.0	2.3978953
7.4	2.0014800	12.0	2.4849065
7.5	2.0149030	13.0	2.5649494
7.6	2.0281482	14.0	2.6390572
7.7	2.0412203	15.0	2.7080502
7.8	2.0541237	16.0	2.7726067
7.9	2.0668627	17.0	2.8332341
8.0	2.0794414	18.0	2.8903847
8.1	2.0918640	19.0	2.9444497
8.2	2.1041341	20.0	2.9957322
8.3	2.1162555	21.0	3.0445437
8.4	2.1282317	22.0	3.0910562
8.5	2.1400661	23.0	3.1354964
8.6	2.1517622	24.0	3.1780715
8.7	2.1633230	25.0	3.2188757
8.8	2.1747517	26.0	3.2581099
8.9	2.1860512	27.0	3.2958495
9.0	2.1972245	28.0	3.3322306
9.1	2.2082744	29.0	3.3672992
9.2	2.2192034	30.0	3.4011974

SIMPLE AND COMPOUND ENGINES COMPARED.

In order to arrive at the merits and capabilities of the compound engine, let us first see what are the results got from a single-cylinder condensing engine of given dimensions and cutting off the steam to work with a certain number of expansions. Let

D = Diameter of cylinder in inches,
L = Length of stroke in feet,
N = Number of revolutions of crank per minute,
p = Mean or average pressure on piston,

then, for arriving at the horse-power we use the following formula:

$$\frac{D^2 \times .7854 \times 2L \times N \times p_m}{33{,}000} \text{ horse-power.}$$

But as $D^2 \times .7854$ = area of piston, and $2L \times N$ = speed of piston in feet per minute, we will make $D^2 \times .7854 = A$, and $2L \times N = S$, the formula then becomes

$$\frac{A \times S \times p_m}{33{,}000} = \text{H. P.}$$

and supposing the cylinder to be 24 in. diameter, length of stroke = 4ft; number of revolutions per minute = 50; pressure of steam at beginning of stroke = 60 lbs. (all pressures here mentioned are above a perfect vacuum), point of cut-off, $=\frac{1}{12}$th part of the stroke, or, after the piston has traveled 4 in., so that

$D = 24$ inches.
$2L = 8$ feet.
$N = 50$ revolutions.
$P_1 = 60$ lbs.

$$p_m = P_1 \frac{1+H}{R} = 17.32, \text{ we get—}$$

$$\frac{452.4 \times 400 \times 17.32}{33{,}000} = 95 \text{ horse-power;}$$

To distribute this power equally over the working parts of a compound engine, it is desirable that both cylinders be so proportioned that they will each give out nearly the same power, and that the thrust caused by the entrance of the steam at the beginning of each stroke be the same in both cylinders.

To attain this with an accuracy sufficient for all practical purposes, it is necessary that the low-pressure cylinder be larger than the high-pressure cylinder in area, by the ratio of expansion that takes place in the high-pressure cylinder; that is to say, if

a = area of piston in high-pressure cylinder,
r_1 = ratio of expansion in high-pressure cylinder,
A = area of piston in low-pressure cylinder, then for equal stroke-lengths

$$A = ar_1.$$

So that if P_1 = initial pressure in small cylinder, and P' = initial pressure in large cylinder, the area of large piston, multiplied by P', will be equal to the area of small piston multiplied by P_1, then $P'A = P_1a$.

But the ratio of expansion will now be the same in both cylinders, and the ratio of the entire expansion in both cylinders is equal to R, the initial pressure in small cylinder, divided by the terminal pressure in large cylinder, hence we get

$$\frac{P_1}{P'} = \frac{P'}{P_2} = r_1, \text{ and } \frac{P_1}{P_2} = R;$$

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and hence $r_1 = \sqrt{R}$ = the ratio of expansion in each cylinder; and as we have already taken $P_1 = 60$ lbs., and $P_2 = 5$ lbs., we have $\sqrt{\frac{60}{5}} = 3.46$ = ratio of areas of the two pistons, and also ratio of expansion in each cylinder, and consequently $= r_1$.

From the nature of the compound engine now considered, the area opened up for the steam by the movement of the large piston is at all times decreased by a proportionate part $= \frac{A}{ar} = 1$, by the advancing area of small piston, so that the space actually occupied by the expanding steam is $= A - 1$, and from this we get the formula for ascertaining the average pressure in the low-pressure cylinder of a compound engine.

$$p'm = P' \frac{H'}{r_2 - 1},$$

where r_2 = ratio of expansion in the large cylinder, and H′ the hyp. log. r_2.*

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* See equation (25).

TABLE C.—SHOWING THE RELATIVE AREAS OF THE TWO CYLINDERS OF A COMPOUND ENGINE, WITH THE AVERAGE PRESSURE IN EACH CYLINDER, ETC.:

P_1	r_1	P'	H'	pm	$p'm$	$p''m$
30	2.449	12.25	.896	23.23	7.52	15.71
35	2.645	13.22	.972	26.10	7.83	18.27
40	2.828	14.14	1.040	28.85	8.04	20.81
45	3.000	15.00	1.098	31.47	8.22	23.25
50	3.162	15.86	1.150	34.00	8.40	25.60
55	3.316	16.58	1.197	36.44	8.58	27.86
60	3.464	17.32	1.242	38.83	8.74	30.09
65	3.605	18.02	1.281	41.13	8.85	32.28
70	3.741	18.70	1.319	43.39	9.08	34.31
75	3.872	19.36	1.353	45.57	9.11	36.46
80	4.000	20.00	1.886	47.72	9.24	38.48
85	4.123	20.61	1.415	49.78	9.34	40.44
90	4.242	21.21	1.444	51.85	9.44	42.41
95	4.358	21.80	1.470	53.84	9.56	44.28
100	4.472	22.36	1.497	55.84	9.64	46.20
105	4.582	22.91	1.521	57.77	9.73	48.04
110	4.690	23.45	1.545	59.69	9.82	49.87
115	4.795	23.98	1.567	61.57	9.89	51.68
120	4.898	24.45	1.589	63.43	9.96	53.47
125	5.000	25.00	1.609	65.32	10.05	55.27
130	5.099	25.50	1.629	67.02	10.13	56.89
135	5.196	26.00	1.647	68.77	10.21	58.66
140	5.291	26.46	1.665	70.51	10.26	60.25
145	5.385	26.93	1.683	72.25	10.32	61.93
150	5.477	27.38	1.700	73.95	10.38	63.57
155	5.567	27.84	1.716	75.67	10.46	65.21
160	5.656	28.32	1.732	77.28	10.52	66.76
165	5.744	28.72	1.748	78.93	10.58	68.35
170	5.830	29.15	1.763	80.55	10.64	69.92
175	5.916	29.58	1.777	82.14	10.70	71.44
180	6.000	30.00	1.791	83.73	10.75	72.98
185	6.082	30.41	1.805	85.32	10.80	74.52
190	6.164	30.82	1.818	86.86	10.85	76.91

The accompanying table has been drawn out for easy reference in conformity with this rule. The first column $=P_1=$ the initial pressure of the steam above a perfect vacuum on entering the small cylinder; the second $=r_1=\sqrt{\frac{P_1}{P_2}}$ shows the relative areas of the two cylinders, and also the number of expansions in high-pressure or in the low-pressure cylinder; the third column, $=P'=$ the terminal pressure in high-pressure cylinder, gives the pressure at beginning of stroke in the low-pressure cylinder; the fourth column, $=H'$ contains the hyperbolic logarithms of r_1; the fifth, $=p_m$, gives the average pressure for a stroke in an equivalent single cylinder, for the different values of r_1 and $=P_1\frac{1+H}{R}$; the sixth column, $=p'_m$, gives the average pressure during a stroke in the low-pressure cylinder of a compound engine, and $=P'\frac{H'}{r_2-1}$; and the last column, p''_m, gives the average pressure for a stroke in the high-pressure cylinder, and $= P_{,}\frac{1+H}{R} - P'\frac{H'}{r_2-1}$.

Now, as the power to be given out by both cylinders is to be the same, the power that is required to be given out by $A = \frac{95}{2} = 47.5$ horse-power, and as $\sqrt{\frac{60}{5}} = 3.464$ $P_2 \times 3.464 = 17.32 = P'$, and from the above formula we get $p'_m = 17.32\frac{H}{r_2-1} = 8.74$ lbs. average pressure per square inch on piston. So that we can now get what area of piston is required to give out this power by

$$\frac{47.5 \times 33{,}000}{400 \times 8.74} = 450 = 24'' \text{ diameter,}$$

and as the area of the two pistons are to each other as 1 to 3.464, we get the area of small piston $= 130$ sq. in. $= 13''$ diameter.

The work performed in one cylinder is here the same as in the other, that is

$$Ap'_m = ap''_m, \text{ or } \frac{A}{a}p'_m = p''_m.$$

But in glancing over the table we see that this relation only holds at $r_1 = 3.464$, the only possible expansion for equalized work, and for equal expansions in the two cylinders.

TABLE D.—OF THE PRESSURE, TEMPERATURE, VOLUME AND MECHANICAL EFFECT OF STEAM.

Absolute pressure in lbs. per square inch.*	Corresponding temperature.	Volume of steam compared with volume of water.	Mechanical effect of a cubic inch of water evaporated, in lbs. raised 1 ft. high.
1	102.9	20868	1739
2	126.1	10874	1812
3	141.0	7437	1859
4	152.3	5685	1895
5	161.4	4617	1924
6	169.2	3897	1948
7	175.9	3376	1969
8	182.0	2983	1989
9	187.4	2674	2006
10	192.4	2426	2022
11	197.0	2221	2036
12	201.3	2050	2050
13	205.3	1904	2063
14	209.1	1778	2074
15	212.8	1669	2086

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* These pressures run lower than those given by Rankine, Cotterill, Zeuner, Rontgen, &c., by 3 per cent. at 30 lbs.; 5 per cent. at 70 lbs; 7 per cent. at 100 lbs., and upward, per square inch.

TABLE D—(*Continued*).

Absolute pressure in lbs. per square inch.*	Corresponding temperature.	Volume of steam compared with volume of water.	Mechanical effect of a cubic inch of water evaporated, in lbs. raised 1 ft. high.
16	216.3	1573	2097
17	219.6	1488	2107
18	222.7	1411	2117
19	225.6	1343	2126
20	228.5	1281	2135
21	231.2	1225	2144
22	233.8	1174	2152
23	236.3	1127	2160
24	238.7	1084	2168
25	241.0	1044	2175
26	243.3	1007	2182
27	245.5	973	2189
28	247.6	941	2196
29	249.6	911	2202
30	251.6	883	2209
31	253.6	857	2215
32	255.5	833	2221
33	257.3	810	2226
34	259.1	788	2232
35	260.9	767	2238
36	262.6	748	2243
37	264.3	729	2248
38	365.9	712	2253
39	367.5	695	2259
40	269.1	679	2264
41	270.6	664	2268

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* These pressures run lower than those given by Rankine, Cotterill, Zeuner, Rontgen, &c., by 3 per cent. at 30 lbs.; 5 per cent. at 70 lbs.; 7 per cent. at 100 lbs., and upward, per square inch.

TABLE D—(*Continued*).

Absolute pressure in lbs. per square inch.*	Corresponding temperature.	Volume of steam compared with volume of water.	Mechanical effect of a cubic inch. of water evaporated, in lbs. raised 1 ft. high.
42	272.1	649	2273
43	273.6	635	2278
44	275.0	622	2282
45	276.4	610	2287
46	277.8	598	2291
47	279.2	586	2296
48	280.5	575	2300
49	281.9	564	2304
50	283.2	554	2308
51	284.4	544	2312
52	285.7	534	2316
53	286.9	525	2320
54	288.1	516	2324
55	289.3	508	2327
56	290.5	500	2331
57	291.7	492	2335
58	292.9	484	2339
59	294.2	477	2343
60	295.6	470	2347
61	296.9	463	2351
62	298.1	456	2355
63	299.2	449	2359
64	300.3	443	2362
65	301.3	437	2365
66	302.4	431	2369
67	303.4	425	2372

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*These pressures run lower than those given by Rankine, Cotterill, Zeuner, Rontgen, &c., by 3 per cent. at 80 lbs.; 5 per cent. at 70 lbs.; 7 per cent. at 100 lbs., and upward, per square inch.

TABLE D—(*Continued*).

Absolute pressure in lbs. per square inch.*	Corresponding temperature.	Volume of steam compared with volume of water.	Mechanical effect of a cubic inch of water evaporated, in lbs. raised 1 ft. high.
68	304.4	419	2675
69	305.4	414	2378
70	306.4	408	2382
71	307.4	403	2385
72	308.4	398	2388
73	309.3	393	2391
74	310.3	388	2394
75	311.2	383	2397
76	312.2	379	2400
77	313 1	374	2403
78	314.0	370	2405
79	314.9	366	2408
80	315.8	362	2411
81	316.7	358	2414
82	317.6	354	2417
83	318.4	350	2419
84	319.3	346	2422
85	320.1	342	2425
86	321.0	339	2427
87	321.8	335	2430
88	322.6	332	2432
89	323.5	328	2435
90	324.3	325	2438
91	325.1	322	2440
92	325.9	319	2443
93	326.7	316	2445

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* These pressures run lower than those given by Rankine, Cotterill, Zeuner, Rontgen. &c., by 3 per cent. at 30 lbs.; 5 per cent. at 70 lbs.; 7 per cent. at 100 lbs., and upward, per square inch.

TABLE D—(*Concluded*).

Absolute pressure in lbs per square inch.*	Corresponding temperature.	Volume of steam compared with volume of water.	Mechanical effect of a cubic inch of water evaporated, in lbs. raised 1 ft. high.
94	327.5	313	2448
95	328.2	310	2450
96	329.0	304	2453
97	329.8	304	2455
98	330.5	301	2457
90	331.3	298	2460
100	332.0	295	2462
110	339.2	271	2486
120	345.8	251	2507
130	352.1	233	2527
140	357.9	218	2545
150	363.4	205	2561
160	368.7	193	2577
170	373.6	183	2593
180	378.4	174	2608

* These pressures run lower than those given by Rankine, Cotterill, Zeuner, Rontgen, &c., by 3 per cent. at 30 lbs.; 5 per cent. at 70 lbs.; 7 per cent. at 100 lbs., and upward, per square inch.

ADVANTAGE OF COMPOUNDING.

From this we can see that for a compound engine to exert the same power as a single-cylinder condensing engine with the same number of expansions in both cases, the low-pressure cylinder of the compound engine requires to be equal in diameter to the single cylinder of the condensing engine; and from this being the case, it seems quite reasonable to say, that if the power exerted can be got from one cylinder with the steam expanded a certain number of times, it would be unwise to add to the expense of the engine by expanding the same number of times in two cylinders instead of one. But as the source of the power obtained is the heat passed into the cylinder from the boiler, and as the economical working of the engine depends on the greatest quantity of this heat that can be converted into power, it is herein where the compound engine has the advantage over any other class of engine, and we will compare the single and the compound engine from this point of view.

CONDENSATION AVOIDED.

The steam enters the single-cylinder engine at a pressure=60 lbs. per sq. in., the temperature of which, on reference to the annexed table, will be found to be equal to 295.6 deg. After doing its work it terminates with a pressure=5 lbs.= 161.4 deg. in temperature, and consequently has cooled down the cylinder to the same temperature. The fresh steam on entering to perform the next stroke can only be effective at a temperature corresponding to its pressure, and it has to part with heat until it brings the cylinder up to its own temperature. This occasions the condensation of a considerable quantity of fresh steam, and consequently the amount to be supplied from the boiler per stroke is increased.

GREATER EFFICIENCY OF THE STEAM JACKET.

Now the pressure at the beginning of the stroke of the high-pressure cylinder of the compound engine is the same= 60 lbs. per sq. in.; but owing to the fewer number of expansions carried out in this one cylinder, it terminates this stroke with a pressure=17.32 lbs.

per sq. in., the temperature of which will be found on reference to be 220 deg., being a difference of only 76 deg. instead of 135 deg., or just about one-half. A great part of this waste of the heat that is passed into the cylinder can be prevented by having a space round about the cylinder, and at both ends, filled with steam at the boiler pressure; but this steam jacket, as it is called, is much more effective in the compound engine than in the single cylinder engine, for this reason: It has been found from experiment that the rapidity with which two volumes of steam of different temperatures seek to equalize themselves is as the square of their difference in temperature, that is to say—that if you mix steam of 200 deg. with steam of 100 deg., and steam of 400 deg. with steam of 100 deg., the difference of temperatures of the former being as 2 to 1, and of the latter as 4 to 1, and as $2^2=4$, and $4^2=16$; the latter temperatures will seek to equalize themselves four times quicker than the former, and as the variation of temperature is much greater in the single cylinder than in either of the cylinders of a compound engine, the heat from the steam jacket must pass through the metal with great rapidity to replace that wasted by condensation, and this it cannot do so effectively as when the temperatures are not so widely varied; and this is one of the great advantages possessed by the compound or double-cylinder engine.

REDUCTION OF FRICTION AND TORSIONAL MOMENT.

Another feature in which the compound engine bears favorable comparison with the single-cylinder engine is in the difference of the thrust caused by the entrance of the steam at the beginning of each stroke, and consequently on the amount of pressure or friction thrown on the crank pin and crank shaft journals, compared with the power to be exerted. If we multiply the area of piston in single cylinder by the initial pressure, we get 452×60=27,120 lbs. total pressure, or equal to a blow of fully 12 tons at the beginning of the stroke. If, in like manner, we multiply the areas of both pistons of the compound engine by their respective initial pressures we get 131×60=7,860 and 452×17.32=7,828, which being added together, gives a total pressure at beginning of stroke when both pistons are moving simultaneously =15,688 lbs=about 7 tons or little more than one-half of that in the single cylinder, and from this it can be easily seen that as a less shock is given to the working parts by about one-half, the dimensions of these parts can be made proportionately less, and a gentler, steadier, but equally effective motion is imparted.

ARRANGEMENT OF PARTS.

The compound engine, both for marine and stationary purposes, has had the position of its cylinders and the combinations of its parts arranged in many different ways, in some cases to suit the space available for its ereticon, and in others, according to the different ideas of the different manufacturers; but the principle being the same in all cases, an equal economy should be got if care is taken in so proportioning the passages for the steam that no undue obstruction is caused, and that proper and efficient means are employed to prevent any waste of heat as far as possible.

The principle of the compound engine was known as early as 1781, when a patent was taken out by Jonathan Hornblower for "employing the steam after it has acted on the first vessel to operate a second time in the other by permitting it to expand itself." But Hornblower was never able to carry the principle to much practical use, owing to the existence of Watt's patents at the time.

THE WOOLF ENGINE AND ARRANGEMENT.

The earliest compound engine in which the principle was practically carried out was patented in 1804 by Arthur Woolf, and his style of engine has been in use almost ever since that date in France and on the Continent generally. It is still constructed by many engineers in this country, and is known as "Woolf's Engine." It is employed for stationary purposes only. Both the cylinders are placed together at one end of the walking beam, the low-pressure cylinder being at the outer end, and the high-pressure cylinder close up to it with a proportionately less stroke. This engine had no intermediate receiver, but the steam was delivered from one cylinder direct to the other through narrow passages. This arrangement is perfectly capable of carrying out the principle equal to any other. But the great pressure it exerts on the main center of the walking beam, owing to all the power requiring to pass from one end of the beam to the other, has caused it to be less extensively used than otherwise might have been the case; but more especially since 1845, when a patent was taken out by Wm. McNaught, wherein this great pressure is removed from the main center by having the low-pressure cylinder only at the outer end of beam, and the high pressure cylinder between the main center and the crank, thus having the power equal on both sides of the main center, and the pressure consequently merely nominal at that part. This arrangement is by far the best when the engine has to be of the walking-beam class.

Another good arrangement is carried out in horizontal engines, with both cylinders lying side by side (generally cast together in one piece), and secured to a single sole plate; both piston rods are attached to one cross-head, so that one connecting rod conveys the whole power to the crank, nothing being in duplicate but the cylinders.

THE WOLFF ENGINE.

In the compound engine at present so largely employed in the British Navy, the cylinders, outwardly, are of the same diameter, but the high-pressure cylinder, as previously mentioned, is generally made about $\frac{1}{8}$ the area of the low-pressure cylinder. The space round the actual high-pressure cylinder is used to receive its exhaust steam, and hence this space is called the receiver. From this it is admitted to the low-pressure cylinder. The pistons do not move simultaneously, owing to the cranks being at right angles to each other in this class of engines. For a treatment of this form of compound engine see *Science Series*, No. 10.

RULES FOR HORSE-POWER.

It has been explained in the first part of this work that the nominal horse-power of an engine is ascertained by assuming the mean pressure on the piston to be equal to 7 lbs. on the square inch, and the speed of piston equal to 220 ft. per minute. But as both the working pressure and speed of piston have been greatly increased since the above rule was first adopted, it fails to convey any adequate idea of the actual capabilities of the engine. Still, in all negotiations connected with the purchase of a steam engine, it is, as a rule the nominal horse-power alone that is referred to, although it is understood that with a pressure, say of about 60 lbs., and a piston speed of about 400 ft. per minute, fully six times the nominal power is got from a condensing engine.

As the term "Nominal Horse-Power" is only used when speaking of the steam engine as a marketable commodity, a particular size of cylinder may be called a certain nominal power by one maker, and a different nominal power by another, and unfair competition often takes place by two manufacturers offering for sale say an 80 horse-power condensing engine, one of whom means to give a cylinder 50 in. diameter, whilst the other calls a 40 in. cylinder the same nominal power. The rules now generally adopted in this country to determine the nominal power of the different kinds of steam engines are as follows:

Rule to find the nominal horse-power of a high-pressure non-condensing steam engine: Square the diameter of cylinder in inches, and divide by 12, that is to say, a non-condensing engine with a cylinder=30 in. diameter, is called a 75 horse-power engine nominal, although it is capable of giving out at least three times the power when a pressure of say 60 lbs. is employed, and piston speed=400 ft. per minute.

Rule to find the nominal horse-power of a single cylinder condensing engine: Square the diameter of cylinder in inches, and divide by 24, that is to say, that a condensing steam engine with a cylinder=30 in. diameter, is called a 37½ horse-power engine nominal, but is capable of working to at least six times its nominal power with 60 lbs. pressure and speed of piston=400 ft. per minute.

The rule now generally adopted by marine engineers for the nominal power of a compound engine is: Add the square of the diameter of each cylinder in inches together, and divide the sum by 30; that is, a compound engine whose condensing cylinder is 30 in. diameter, and high-pressure cylinder 17 in. diameter, is called a 40 horse-power compound engine nominal. But it is capable of working to at least six times that power with

60 lbs. pressure and speed of piston= 400 ft. per minute.

INDICATOR DIAGRAMS.

Some diagrams (Figs. 1 to 8) are herewith given, the first two of which are theoretical, and the shape that would actually be got was there no loss of heat during the stroke from condensation or other causes. In the theoretical diagram, (Fig. 1), showing the expansion curve when the steam is expanded 12 times in a single cylinder condensing engine, AB represents the total initial pressure of 60 lbs., BC the constant supply of steam from the boiler at that pressure, C the point where the steam is entirely shut off=$\frac{1}{12}$th part of the stroke, CD the expansion curve formed by the decreasing pressure of the steam in the ratio that the space it occupies is increased by the advance of the piston, DE represents the terminal pressure, and EA the line of perfect vacuum. In (Fig. 2) the compound theoretical diagram, CD is the expansion curve formed from the high-pressure cylinder, and DA the expansion curve formed from the condensing cylinder, the line FB representing the initial pressure of 60 lbs., DE the terminal pressure in high-pressure cylinder, and initial pressure in low-pressure cylinder, and equal to 17.32 lbs., and FA the terminal pressure in low-pressure cylinder, and equal to 5 lbs.

Fig. 1.—*A Theoretical Diagram showing the Expansion Curve in a Single Cylinder Condensing Engine, with Steam at 60 lbs. pressure above a perfect vacuum, expanded 12 times.*

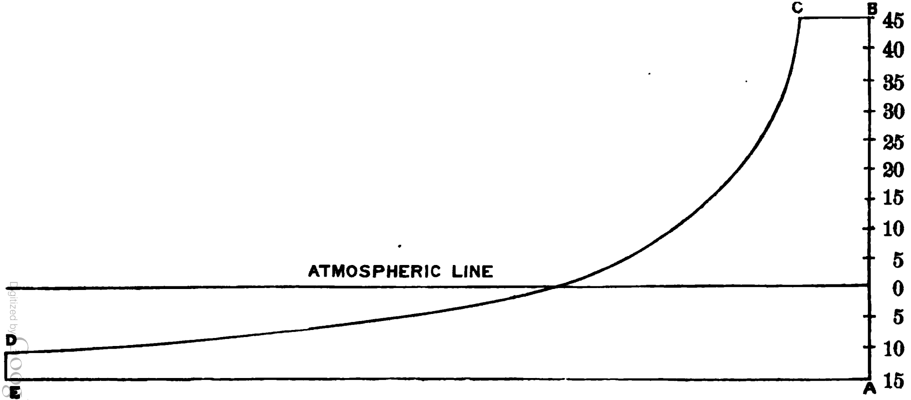


FIG. 2.—*Theoretical Diagrams showing the Expansive Curve in both Cylinders of a Compound Engine, with Steam at 60 lbs. pressure above a perfect vacuum. Total number of expansions*=12.

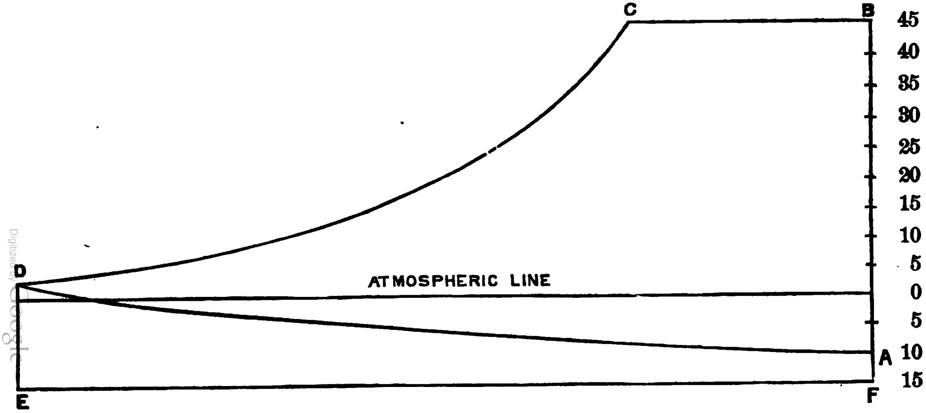
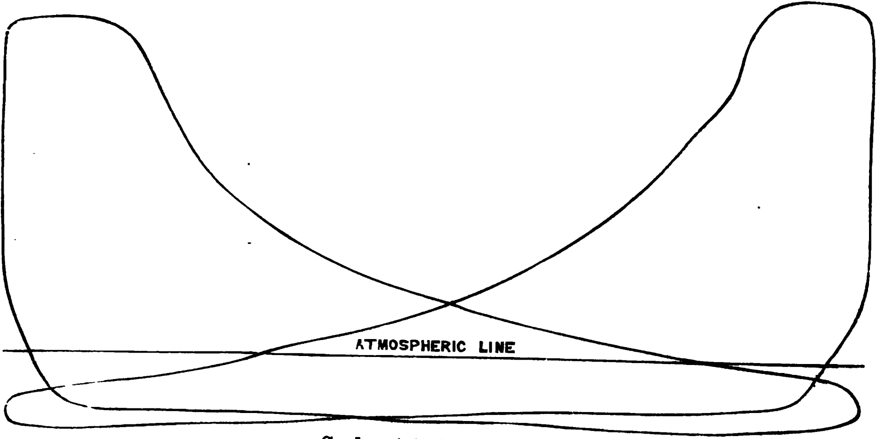
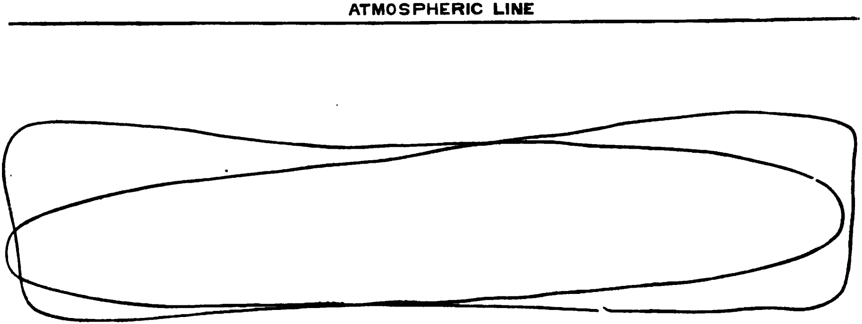


FIG. 3.—HIGH PRESSURE.—*Diagrams taken from a Compound Engine with Steam cut-off after the Piston has travelled one-eighth of the stroke in high-pressure Cylinder.*



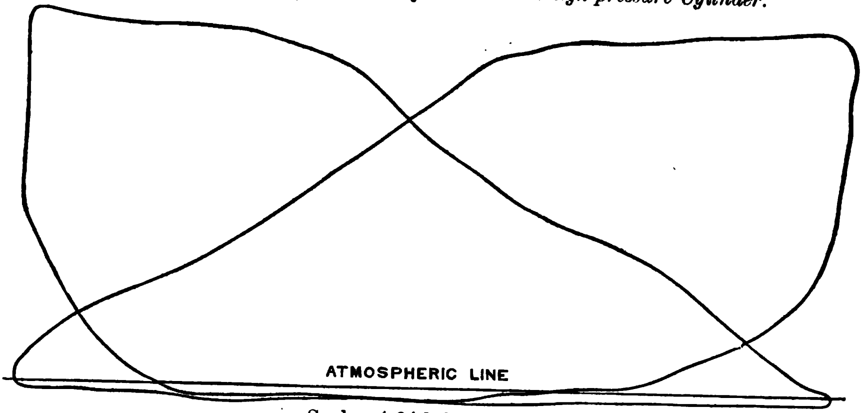
Scale—1-16 inch=1 lb.

FIG. 4.—LOW PRESSURE.—*Diagrams taken from a Compound Engine with Steam cut-off after the Piston has travelled one-eighth of the stroke in high-pressure Cylinder.*



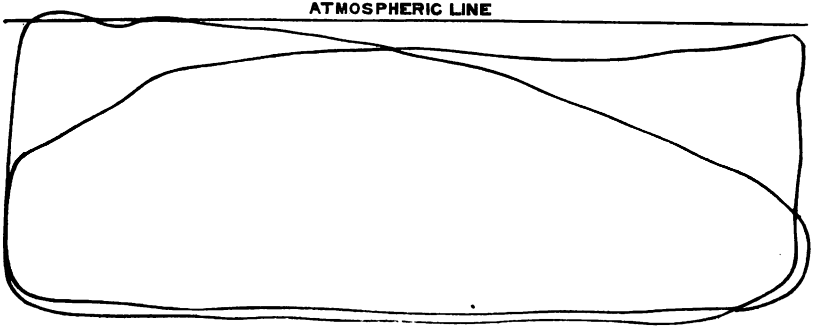
Scale—1-8th inch=1 lb.

FIG. 5.—HIGH PRESSURE.—*Diagrams from the Compound Engine of the S. S. "Dunluce Castle'—Steam cut-off at one half the stroke at high-pressure Cylinder.*



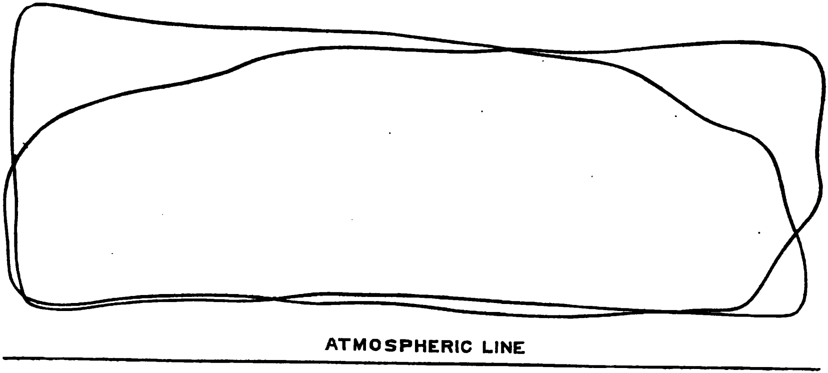
Scale—1-24th inch=1 lb.

FIG. 6 —LOW PRESSURE.—*Diagrams from the Compound Engine of the S. S. "Dunluce Castle"—Steam cut-off at one-half the stroke in high-pressure Cylinder.*



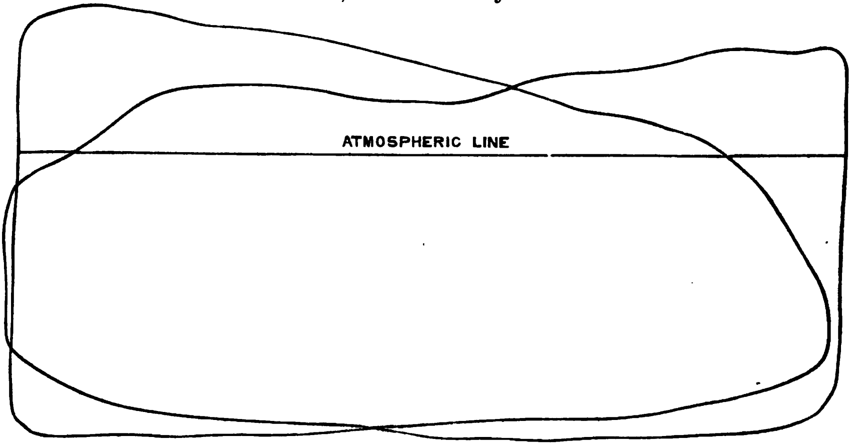
Scale—1-8th inch=1 lb.

FIG. 7.—HIGH PRESSURE.—*Diagrams from the Compound Engine of the S. S. "Dunluce Castle," with the Link full out.*



Scale—1-24th inch=1 lb.

FIG. 8.—LOW PRESSURE.—*Diagrams from the Compound Engine of the S. S. "Dunluce Castle," with the Link full out.*



Scale—1-8th inch=1 lb.

PART II.

Graphical and Analytical Treatment,

ILLUSTRATED WITH DIAGRAMS, TABLES, ETC.

INCLUDING

ROTATIVE

AND

PUMPING NON-ROTATIVE ENGINES.

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THE TWO-CYLINDER COMPOUND ENGINE

IN WHICH THE
STROKES ARE SIMULTANEOUS, OR CO-INITIAL
AND CO-TERMINAL, WITH RECEIVER,
CUSHION, CLEARANCE, ETC.

THE reader has doubtless observed that the preceding part of this little volume treats only of the Woolf compound engine, in which there is no receiver and where the high and low-pressure cylinders are in immediate steam connection, without even allowance for communicating passages, or clearance, between the cylinders; and that the pistons make simultaneous movements throughout their strokes: Also that the steam is cut off only in the high-pressure cylinder, and that the work developed in the two cylinders is made about equal.

The following has been added for the purpose of making the treatment more complete and general for this particular type of compound engine, and to inclnde the intermediate receiver, clearance and cushion.

Probably the simplest form for this engine is that one much in use for pumping engines, and illustrated in the accompanying cut, Fig. 9, of a Worthington pumping engine. A is the high-pressure cylinder, B the low-pressure cylin der and C the condenser.

Steam is admitted directly to A from the boiler; from A it is exhausted through the pipe shown partly obscured by the steam chests, directly to the large cylinder B, and from thence it is taken to the condenser.

The piston rods of A and B are both connected with one cross-head. When the pistons are both moving toward the left, high steam enters at the right-hand side of the piston A, while from the left side the steam passes freely through the connecting pipe to the right hand side of the piston B, the left side of B being open to the condenser.

It is evident that the pump might be removed and a crank and pitman substituted. The engine then becomes "rotative," that is, has a rotating shaft.

It is not essential that the pistons both move in the same direction at the same instant in a rotative Woolf engine, as would be the case in Fig. 9 made rotative; but one cylinder might be connected up with one crank, and the other cylinder with a second crank on the same shaft placed at 180° with the first crank. The pistons would then move simultaneously in opposite directions with strokes co-initial and co-terminal in points of time.

Or instead of placing the cylinders both on one side of the shaft, they might be placed on diametrically opposite sides of the shaft. Then both could be connected to the same crank, or two cranks at 180° could be put one to each cylinder.

Again, the cylinders could be arranged at quartering points and the cranks likewise.

For the cylinders both at one side, many ingenious combinations of cylinders have been devised, some being of the trunk engine form, etc.

One arrangement is theoretically as economical as another, except for considerations of waste heat, and for large intermediate communicating pipes acting like receivers for remote cylinders. The best arrangement for a minimum of intermediate passage is where the cylinders are placed side by side, separated only by valves and acting on cranks at 180°. Then one piston moves opposite to the other and the steam passes over direct through short passages.

When the pistons are connected to cranks in such manner that the strokes are not co-initial and co-terminal, a considerable volume is required in an intermediate receiver. The engine then becomes the Wolff engine, the special considerations relative to which will not be entered upon here, but will be found treated in *Science Series*, No. 10.

In the study of any form or type of

engine, the correct form of the diagram of the action of the steam in its passage through the cylinders is important; it must precede formulas, because formulas cannot be established until the steam action is perfectly understood. We will now attempt to trace the diagram for any form of the Woolf engine.

In Fig. 1 of the first part of this book, we have a theoretical diagram of a simple one-cylinder, expansive condensing engine. The line AB is the axis of pressures or the line of zero volumes, and AE the axis of volumes or the line of zero pressures. To hereafter express this, we will only speak of the point A as the zero of pressures and volumes. On the supposition that the steam expands, according to the law of Mariotte, the expansion curve CD is a common hyperbola, such that the product of a pressure by its corresponding volume is equal to the like product for any other point.

A very convenient way for tracing this curve on a drawing board is as shown in

Fig. 10. Take O as the zero of volumes and pressures. Then, suppose *a* to be any point in the expansion curve. Draw any straight line *abcd*, and make $cd = ab$. Then *c* is a point in the curve required. Then draw a second line through *c* and make $ab = cd$, and *a* is a second point in the curve. Any number of lines may be similarly drawn to find a like number of points. Thus, when any one point is given, as, for instance, C or D, Fig. 1, any other points may be easily located through which to trace the curve CD. See also *American Machinist*, Dec. 16, '82; Wm. Lee Church.

But the actual steam expansion curve differs from Fig. 1. It is apt to be steeper at the portion near C, and flatter toward D. This is explained partly on the ground of partial condensation in the cylinder at the initial expansion, and of re-evaporation near the end of the expansion. In the average actual indicator diagram an hyperbola through C will very nearly strike D, for which reason this curve is mostly used in the examination of diagrams traced by the indicator The position of the actual curve, how-

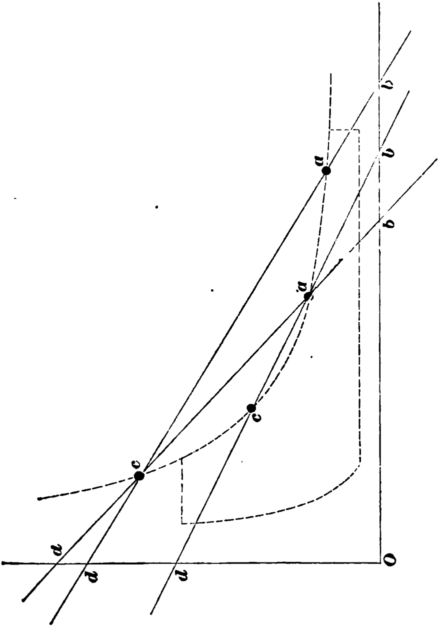

FIG. 10.—TO DRAW THE EXPANSION CURVE.

ever, will vary much with circumstances, and usually the higher the speed the lower will be the point D for a given point C. Expansion lines can therefore be drawn with a degree of approximation with the simple method of tracing given in Fig. 10.

Fig. 2 represents a theoretical indicator diagram for a Woolf compound engine. The part ABCDA is for the high-pressure cylinder; while the part ADEFA is for the low-pressure cylinder. The latter is very much smaller than the other; accounted for by the fact that the pressure is much lower in the low-pressure cylinder than in the other, while the length is made the same by the indicator. In studying these diagrams it is convenient to increase the length of the low-pressure diagram so that it is as many times longer than the high-pressure diagram as the low-pressure cylinder is larger in volume than the high-pressure cylinder.

Then if the areas of the high and low pressure diagrams are equal, the work developed by one cylinder equals that by the other.

To thus enlarge the low-pressure diagram and put the resulting combined diagram into the best shape for study, horizontal lines should be drawn through the diagram ADEF and increased in length by the ratio of the cylinder volumes as was first done by Rankine and as indicated in Fig. 11. Take AE = the volume of the high-pressure cylinder, and AF = volume of the low-pressure cylinder. Then, as in Fig. 2, BCDH is the indicator diagram of the high-pressure cylinder, and AHDE of the low-pressure cylinder as taken by the indicator itself from the engine. Draw LKJ and make LJ = LK × ratio of cylinder volumes. Then J is a point in the new curve; any number of points in DJG may be thus found. Then BCDJGFA is the combined diagram, and the work developed per stroke of engine will be represented by it, also the work developed in each cylinder will be represented by its respective part of the area BDCH or

AHDGF. The area AHDGF is equal to the area AHDE × ratio of volumes of cylinders as appears from the fact that in the transformation of this diagram the horizontal dimensions only are altered, and these in the ratio of cylinder volumes. This diagram supposes no clearance, no dead space between the cylinders, no cushioning, and no resistance to the motion of the motor fluid.

I.—DIAGRAM OF A COMPOUND ENGINE WITHOUT RECEIVER OR CLEARANCE, BUT WITH HIGH-PRESSURE CUT-OFF.

The action of the steam in passing through the engine whose diagram is presented in Fig. 11, may be thus traced, viz., BC is the admission line of steam at boiler pressure. At C the admission is arrested by a proper cut-off, when expansion goes on till the end of the stroke AE is reached for the high-pressure cylinder. The expansion curve CD has the point A for the zero of volumes and pressures; that is, for hyperbolic expansion the curve CD is to be constructed by aid of the lines AB and AF as explained in Fig. 10 from the lines A*b* and A*d*.

At the end of the stroke of the high-

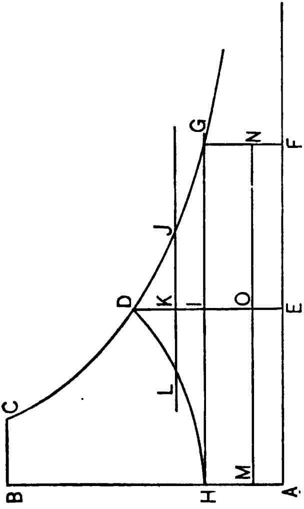

FIG. 11.—COMBINATION OF DIAGRAMS.

pressure cylinder the volume of steam admitted has become AE the volume of the high-pressure cylinder. Now a valve opens a passageway between the cylinders so as to discharge this steam from the full high-pressure cylinder into that end of the low-pressure cylinder at which the stroke is just beginning; but as there is no clearance, &c., this opening of the valve makes no change in the total volume of steam, the volume being still AE. Now the pistons are simultaneously to move and make their strokes, the steam to flow from the high-pressure cylinder into the low-pressure cylinder. In this act the volume enlarges from that of the high to that of the low-pressure cylinder; that is, from AE to AF, Fig. 11, and the pressure of this expanding steam is the same at any instant in the two cylinders it being a back-pressure in the high and a forward-pressure in the low-pressure cylinder. Taking DG as the expansion line for this change of volume, AE to AF, we would have some line DLH of back-pressure for the high-pressure cylinder. This line begins and ends at the same height as the expansion line DG, in fact the lines throughout are at the same height at corresponding points L and J. At these points KL is the part of the stroke made by the high-pressure cylinder, LJ the part of stroke of the low-pressure cylinder, while JK is the corresponding increase of volume of the steam. The line MN may represent the back-pressure line of the low-pressure cylinder it being the line of pressure of the atmosphere or of the condenser.

Now in the expansion line DG the steam expands from the volume AE to the volume AF, just as though the first or high-pressure cylinder had been long enough to have carried the expansion to that extent. From this it appears that the curve DG is simply the continuation of the curve CD, all drawn from A as the zero of volumes.

From this there appears to be no gain from compounding, because the diagram is the same as though the admission and expansion had all taken place in the low-pressure cylinder, itself and alone. The advantage in compounding is to be found outside of the diagram and will be considered elsewhere.

II.—DIAGRAM OF A COMPOUND ENGINE WITH NO CLEARANCE OR CUT-OFF, BUT WITH RECEIVER.

Let the volume of the high-pressure cylinder be BC, of the low-pressure cylinder be BD, and of the receiver AB. Now, supposing the engine in continuity of action, the diagram may be traced as follows: Represent the admission for full stroke by FL. At the end of the stroke the valve between the high-pressure cylinder and receiver opens when the steam in these parts commingle and the pressures equalize, the cylinder steam expanding and the receiver steam being compressed. As LF is the volume of the cylinder steam, this expansion will be along an expansion line FJ with the point C as the zero of volumes. The receiver steam will have the volume AB, and its pressure will be BH=DN=the terminal pressure of the low-pressure cylinder. Now as the "fresh" steam FL is introduced, this receiver steam will be compressed along a com-

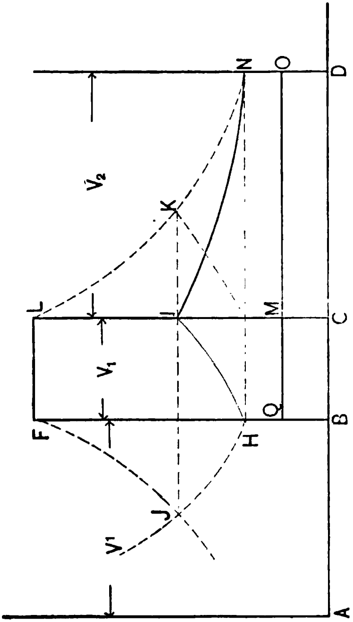

Fig. 12.

pression line HJ with A as the zero of volumes. The point J of intersection of these curves gives the resulting pressure of the combined steam. The curve MK =HJ, giving K=J, might have been used. This pressure is that with which the low-pressure cylinder begins its stroke; and it should be laid off on the line at CI, because now the volume of steam is AC filling the receiver AB, and high-pressure cylinder BC. Now during the stroke this steam changes volume from AC to AD, because the high-pressure cylinder ejects its steam into the receiver, thus giving cause for a reduction of volume from AC to AB, while at the same time the low-pressure cylinder gives cause for an increase to AD, the difference being CD, the actual increase as stated. The expansion curve IN will be drawn with A as the zero of volumes, because AC is the lesser and AD the greater of the limiting volumes. The back-pressure line IH to the high pressure cylinder will correspond in height with IN, as explained in Fig. 11. The back-pressure line to the low-pressure cylinder is to be accounted for as to whether the exhaust is into a condenser or into the atmosphere.

Detrimental Effect of the Receiver.—This as a steam diagram differs from that of Fig. 11 by a depression of the expansion line IN from the dotted position LN. This engine with its receiver is thus seen to be less efficient than that of Fig. 11 by the absence of the area ILN. The smaller the receiver the less the depression or "drop" of the point I. For $AB=o$, I coincides with L. For $AB=$ infinity the line IN is horizontal and straight. But the point N is fixed, its position being entirely independent of the volume of the receiver, and will always be found on the expansion line LN, the latter being drawn with B as the zero of volumes.

This important truth is apparent from the fact that at the end of the stroke of the low-pressure cylinder we discharge just the cylinder full of steam, the weight of which is the same as the weight of the steam admitted per stroke. Assuming the latter the same for two cases, one having a receiver and the other not; then it follows that the weight of the low-pressure cylinder full of steam exhausted per stroke is the same for both cases, so that if the small cylinders are of equal size the pressure of the contained steam at stroke terminations is the same for both; and also if the large cylinders are of equal volumes the pressures of their contained steam will be the same in both, thus making the points L and N coincide in both cases, both points being found on the line LN drawn with B as the zero of volumes and pressures as stated. This fact gives a ready means for locating the point N in tracing a diagram.

Comparison with an Indicator.—The diagrams taken directly by the application of an indicator to the engine will be HFLI for the high, and QHIM for the low-pressure cylinders. The relation of the work performed by the two cylinders can be obtained by measuring the areas just named with a planimeter, multiplying QHIM by the ratio of volumes and comparing with HFLI. But when the low-pressure diagram is expanded to BD, as in Fig. 12, the areas, and consequently the work of the two cylinders can be compared by sight, or by planimetric measurements.

The diagram that would be obtained by use of only the low-pressure cylinder as a simple engine, would be QFLNO, so that according to the diagrams the simple engine would be more efficient than the compound of Fig. 12 by the ratio

$$\frac{\text{QFLKNOQ}}{\text{QFLINOQ}}$$

Re-heating.—The receiver, which is responsible for the drop LI, appears by the diagram to be a detriment. This drop may, however, be more than compensated by using the receiver for a "re-heater" of the steam, by passing high steam through it in tubes. The intermediate steam may thus be raised in temperature to nearly that of the boiler steam, and consequently be somewhat superheated and made to work dry. The whole line IN may thus be raised to an extent nearly proportional to the elevation of the *absolute* temperatures of the steam by superheating, the absolute zero being a point about 493° F. below melting ice. (See lower part of Fig. 22.)

A small intermediate passage, such as shown in Fig. 9, will have the effect of a small receiver, and cause its corresponding drop LI.

The diagram Fig. 12 supposes no cut-off to either cylinder. There may, however, be one to both, as in the next case.

III.—DIAGRAM OF A COMPOUND ENGINE WITH A CUT-OFF TO EACH CYLINDER, AND WITH AN INTERMEDIATE RECEIVER.

In Fig. 13 take BC = volume of high cylinder, BD = B′D′ = volume of the low cylinder. AB = volume of the receiver, E = point of cut-off for the high cylinder, and N = the point of cut-off for the low cylinder. Also G′ = the point of release of the steam from the low-

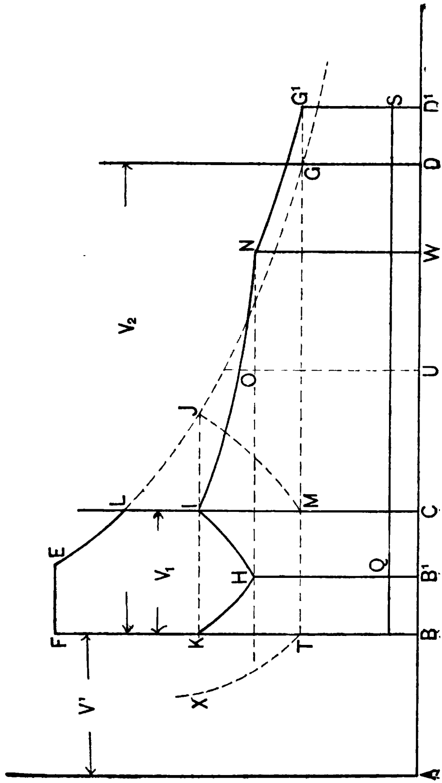

FIG. 13.

pressure cylinder, and QS = the back-pressure line of the final exhaust.

Now admit boiler steam FE, and cut-off at E, in the high cylinder BC. This steam expands along the expansion line EL, drawn with B for the zero of volumes and pressures. At the stroke terminal L, the valve between the high cylinder and receiver opens and the steam of these parts commingle; that of the former expanding along the line LJG with B for the zero of volumes, and of the latter along the compression line TX, with A for the zero of volumes. By turning over the curve TX, and transferring it to MJ, we obtain the intersection J, the height of which point above AD gives the resulting pressure of the commingled steam. A horizontal line through J gives I, the point from which lines of further action are to be drawn.

At this point of time the high cylinder piston is at the end of stroke, the low cylinder piston at beginning of stroke, and the volume of steam under action is that of AC, which fills the receiver and high cylinder in common. In the abscence of a low cylinder cut-off, the stroke of pistons now beginning would expand the steam from its present volume AC to the volume AD, and the pressure of release would be DG, the point G being on the linè ELJ prolonged, as explained in Fig. 12. But suppose the low cylinder cut-off occurs at half stroke. This cut-off increases the average receiver pressure, so that the expansion line IN will now cut the line ELG and place N outside. To trace the curves G′NIHK, we find that the expansion line IN will be drawn from I, with A for the zero of volumes. To find the point N, suppose the cut-off in the low cylinder occurs at half stroke. It will also be at half stroke of the high cylinder, and the expanding volume between pistons at the instant of this cut-off will be the receiver AB + half the cylinder BC + half the cylinder BD. Take $BU = \frac{1}{2}$ BD, and $UW = \frac{1}{2}$ $BC = BB'$. Then N will be found at the point where $AW = AB + BU + UW =$ the total volume expanding from I with the zero of volumes constantly at A.

Now when the steam is cut off at N, and at half stroke, the volume inclosed in the low cylinder will be the half of that cylinder=B′W. Hence the expansion will go on from N to G′ with B′ for the zero of volumes until we have WD′ $=\frac{1}{2}$ BD$=\frac{1}{2}$ B′D′ completing the remaining half stroke. The line IH will be in common between the two diagrams from the same causes, as is DH, Fig. 11, or IH, Fig. 12, in common for those diagrams. Drawing from the half stroke point H, the line HB′, and also the back-pressure line QS, we complete the theoretical diagram for the low cylinder, HING′SQH. When the cut-off N occurs there is shut up in the receiver and the high cylinder the volume AB+BB′. The remaining stroke of the high cylinder will compress this to AB, giving the compression line HK, with A for the zero of volumes. Hence the high cylinder diagram is FELIHKF.

Effect on Distribution of Work of the Low Cylinder Cut-Off.—Were there no cut-off to the low-pressure cylinder, the expansion in this cylinder would terminate at G instead of at G′, and start from some point below I. From this it appears that the low cylinder cut-off has the effect to raise the mean pressure in the low cylinder and diminish it in the high cylinder. Hence, by applying a cut-off to the low cylinder we may increase the work performed in that cylinder, and diminish that performed in the high-pressure cylinder.

The terminal pressure D′G′ will be the same as though there were no cut-off to the low-cylinder, because the steam admitted is FE, and it must occupy the same volume B′D′=BD=the low-cylinder full at release, whether there be a low cylinder cut-off or not. Hence the terminal pressure D′G′=DG=CM will be

$$D'G' = BF \times \frac{FE}{BD}$$

if the Mariotte law of expansion is assumed to hold. By thus drawing the diagram Fig. 13, the relative or total areas or work of cylinders may be obtained.

To draw IH, take a zero of volumes to the right of C a distance

$$=AC\frac{V_1}{V_2-V_1}$$

In the present case, when the receiver is very large, the line KHIN comes to be nearly horizontal. and is quite so for an infinite receiver. When the receiver is zero in volume, the cut-off N must be dispensed with, and then the point I will rise to L.

IV.—DIAGRAM OF A COMPOUND ENGINE WITH A CUT-OFF TO EACH CYLINDER, WITH A RECEIVER, AND WITH CLEARANCE AND CUSHION.

In Fig. 14 take $AB_1=V'$ = volume of the receiver, $BC=V_1$=volume of the high cylinder, B_1B = volume of clearance to the high cylinder, B_1B_2 = volume of clearance to the low cylinder, and $B_2D=V_2$=volume

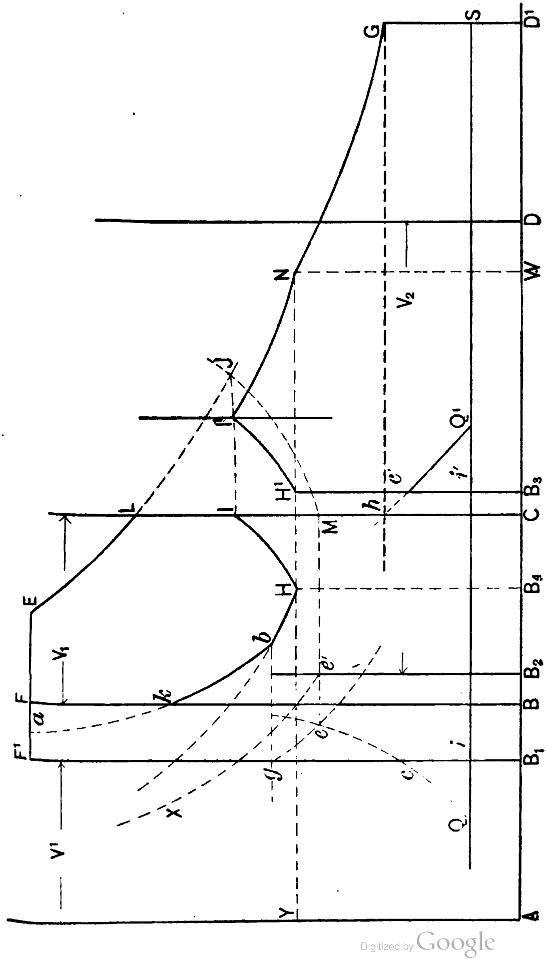

FIG. 14.

of the low cylinder. Also in the high cylinder let E be the point of cut-off, L the release, IH the back pressure line for $\frac{3}{8}$ the stroke or till the cut-off occurs in the low cylinder, H*b* the cushion line against the receiver, and *bk* the cushion line against the clearance; and for the low cylinder, let I′N be the combined expansion line preceding cut-off at $\frac{3}{8}$ stroke, NG the expansion line subsequent to cut-off, SQ′ the back pressure line, Q′*c*′ the cushion against clearance, and H′I′ the counterpart of I′N. To better fix the ideas, let Fig. 15 represent the main features of the compound engine now considered. A being the high cylinder, B the low cylinder, C the receiver, F the boiler supply, G the exhaust, D the valves for A, and E the valves for B. The receiver includes all the space between D and E, the clearance of one end of A, all the space between D and the piston displacement for that end of A, and likewise the clearance of B, all the space between E and the piston displacement of B.

Now supposing the engine in continu-

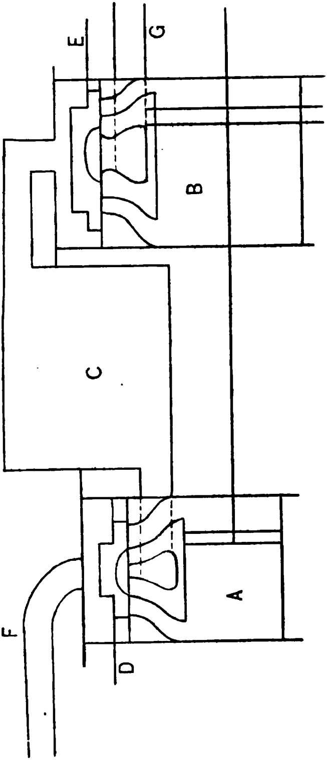

FIG. 15.

ity of action, let steam be cut off at E, Fig. 14, the clearance F′F, of course, being full of steam also. The expansion for the remainder of stroke will give the expansion line EL, drawn with the bottom of the clearance B_1, as the zero of volumes.

This is due to the fact that F′E and not FE is the original volume of the steam now expanding. At L, the high cylinder and its clearance are opened by a valve into the receiver, and the steam of these parts commingle, that of the former expanding along a line LJ, and of the latter being compressed along the line $e'X =$ MJ; both taking a common pressure J at the intersection. The point M is understood in this to give the pressure in the receiver and low-cylinder clearance at the instant the stroke is ended; and the valve between receiver and low cylinder is opened. The pressure M will be explained further.

Drawing a horizontal line through J we obtain I for the initial back pressure line IH of the high cylinder V_1; and I′ for the initial forward pressure of the low cylinder V_2, I' being distant from I by the clearance of V_2, viz. B_1B_2. $I'H'$ is parallel to IH, that is, $II'=HH'$. We are now at the point of beginning of another stroke which will force the remaining steam from V_1 and fill V_2.

At this point of time the total volume of steam considered is that of the receiver, that of the high cylinder and its clearance, and that of the clearance of the low cylinder. It is $AB_1+B_1B+BC+B_1B_2=$ a volume extending to the point I' where $II'=B_1B_2$. This volume will now expand along an expansion line $I'N$ with A for the zero of volumes.

The point N is found by laying off the total volume for the time of cut-off in the low cylinder which has been taken above at $\frac{3}{8}$ the stroke. This fraction being the same for both cylinders we have H at $\frac{3}{8}$ stroke from I, so that $YH = AB_1+B_1B+\frac{3}{8}BC=$ receiver + high cylinder clearance + $\frac{5}{8}$ high cylinder volume. To this add the low cylinder clearance $B_1B_2=HH'=II'$; and also add $\frac{3}{8}$ the volume of the low cylinder, or $\frac{3}{8}$ B_2D and we have the total volume $YH+HH'+H'N=YN$, $H'N$ being equal to $\frac{3}{8}$ V_2. Beyond N the steam inclosed in the low cylinder will expand the remaining $\frac{5}{8}$ of stroke, giving the expansion line NG, drawn from B_4 as the zero of volumes; where NG, or $D'W=\frac{5}{8}$ the stroke, NH′, or $WB_3=\frac{3}{8}$ of stroke, and H′H, or $B_3B_4=$clearance.

The cushion line $Q'c'$ is due to the compression of a portion of the exhaust steam into the clearance, and is found by drawing Qc with B_2 as the zero of volumes, where $Qi=Q'i'$. The piston will compress the steam to c when further compression will result from the reversal of the valve and the commingling with it of the steam from the receiver. The pressure of this latter steam now admitted from the receiver is found from the point b where the high cylinder closes its valve for cushion, and after which the receiver is abandoned to itself till the end of the stroke is reached. This pressure b is B_1g, and as the steam is thus admitted from the receiver the pressures B_1c and B_1g combine at the intersection e of the line Qce drawn from B_2 as the zero of volumes, and the line ge drawn from A as the zero of volumes. The total volume now is AB_2 and pressure B_2e', so that the line $e'X$ of further compression can be drawn with A as the zero of volumes. Transferring this to MJ, and prolonging EL to J, we have the point of intersection J as that at which the steam now released from the high cylinder, combines with that in the receiver to a common pressure.

In the high cylinder and receiver there will be compression Hb, the curve being drawn with A as the zero of volumes. At b the valve between receiver and high-cylinder is supposed to close giving a compression line bk, which may be produced to a, drawn from B_1, as the zero of volumes.

The cushioning in the cylinders fills part of the clearances so that the steam admitted per stroke is aE at the pressure BF, and that exhausted is hG at the pressure D′G, the curve $Q'c'h$ being understood to be the same as Qce. The last statement enables us to find the pressure D′G; and it is=

$$D'G = BF \cdot \frac{Ea}{Gh}$$

according to the Mariotte law of expansion.

For convenience in drawing the curve H′I′ we observe that the change of volume H′I′ is $\frac{3}{8}$ V_1, and the change of volume I′N is $\frac{3}{8}$ $(V_2 - V_1)$, so that H′I′ may be drawn from a zero of volumes taken at a distance to the right of B_3=

$$AW \frac{V_1}{V_2 - V_1}.$$

The larger the receiver the more nearly horizontal becomes the line *b*HII′N, it being quite horizontal for an infinite receiver. On the other hand, the smaller the receiver the steeper become the declivities of that line and the more nearly I approaches to L, and the latter should be the cut off N. But I never coïncides with L while clearance exists.

The cut-off N may be made useful in

equalizing the amount of work performed by the two cylinders.

RELATIVE EFFICIENCIES.

In all the foregoing diagrams the theoretical diagrams have been sought, such as are needed for studying the relative efficiencies of engines, and not such as would be obtained by the indicator. Of several engines of different volumes of receiver, clearance, cushion, &c., of which the diagrams have the same heights BL, and the same steam volumes Ea; that engine which shows the greatest added area of both diagrams has the highest efficiency. Such efficiency can be investigated graphically as above, and that method is probably the most practical, because analytical formulas for such cases as the last two above, become so excessively complex as to make more trouble and labor than will the drawing board and planimeter.

DOCTORING OF THE DIAGRAMS FOR SPEED, CONDENSATION, RE-HEATING, ETC.

The faster the engine runs and the greater the amount of work done the greater will be the resistance to the fluid through the passages from the boiler through the engine to the condenser or atmosphere. This resistance causes a lowering of the admission line FE, a raising of the exhaust line Q′S, and a vertical separation of the lines IHbk and H′I′NG, of Fig. 14, and likewise for the other cases. An approximation to the amounts of these effects can be obtained by the aid of the formulas for flow through orifices and pipes, which are,

For orifices

$$v^2 = \frac{q^2}{a^2} = 2u^2gh = 64u^2h \text{ nearly,}$$

and for pipes or passages

$$v^2 = \frac{q^2}{a^2} = \frac{2gha}{f's l} = 1100.\frac{ha}{sl} \text{ nearly;}$$

where the head h may be obtained from

the pressures and density of steam or air by the formula

$$h=\frac{p'-p''}{\delta}$$

In these formulas v=velocity of fluid through orifice or passage, q=the cubic feet of fluid per second, a=area of cross section, orifice or passage, s=perimeter of same section, l=length of passage, all in feet, u = a coefficient of velocity through orifice,=0.82 for orifice of entry to a tube, or=0.62 for an orifice through a thin partition, f=coefficient of friction=about 0.006, $p'-p''$=the fall of pressure lbs. per square foot in going through the orifice or passage, and δ=the density or weight per cubic foot of the flowing fluid. Its value may be obtained from Table D by dividing the weight of a cubic foot of water (about 62 lbs.) by the value in the volume column of table corresponding to the pressure for which δ is sought.

The formulas apply only for comparatively small falls of pressure, such as that from boiler to steam-chest, or from steam-chest to cylinder.

In selecting the value of δ, care should be taken to find its true value. In wet steam the moisture should be included. From this cause the value of δ may rise to two, or even three times its value for dry steam.

If w = velocity of piston in feet per second, and A = area of piston in square feet, we have

$$q = \mathrm{A}w$$

for steam passages leading to and from cylinders. Introducing this, and taking u at .8, also eliminating h, we obtain the practical formula for entry to passages, and for short passages

$$\frac{\mathrm{A}^2w^2}{a^2} = 41\,\frac{p'-p''}{\delta}$$

and for passages only,

$$\frac{\mathrm{A}^2w^2}{a^2} = 11000.\frac{a}{sl}.\frac{p'-p''}{\delta}.$$

Combining the two formulas into one for application to passages whose lengths vary from 10 to 300 or 400 times their thickness, we obtain

$$\frac{A^2w^2}{a^2}\left(1+.0074\frac{sl}{a}\right)=41\frac{p'-p''}{\delta}$$

It is generally best to assume a value for the "*a*" in the parenthesis of the last formula.

The rounding of the corners of the diagrams can only be done by the judgment of the engineer. The greater the speed of the engine the earlier must be the release, in order that the release end of the diagram shall not be too sharp at the top corner at the expense of excessive curtailing of the lower corner. The rounding of the cut-off corners must be done with reference to both the speed of engine and rapidity of closure of cut off valve.

Considerable condensation is liable to occur in the high-pressure cylinder when it has an early cut-off, so that the expansion line for it may fall with undue rapidity. This may be avoided in a measure by use of the steam jacket. The condensed steam flowing from this cylinder will accumulate in the receiver unless disposed of by escape traps, or by reheating, the latter process consisting of placing hot surfaces in or about the receiver. These heating surfaces may be tubes passing through and containing high steam from the main boiler, or from a second boiler, which may be at even a higher temperature and pressure than the steam in the first named one. The steam may thus become superheated for the low-pressure cylinder. If now no water has been drawn off, but if all the contents from the high-pressure cylinder is changed into superheated steam, then the expansion curve for the low-pressure cylinder may first be drawn on the supposition of dry steam, and then to be raised nearly to the extent of the elevation of the absolute temperature of that steam above the temperature due to its pressure as dry steam. (See Fig. 22). The latter temperature is given by tables of the properties of steam. (See Table D in the first part of this book. If any condensed steam is trapped out, the expansion curve of the low-pressure cylinder must, provided the steam admitted be dry, be below that due to dry steam.

PRECISE FORM OF THE EXPANSION AND COMPRESSION CURVES.

In a diagram such as that of Fig. 13 or Fig. 14, the steam curves differ in character. From the standpoint of thermodynamics, the curve EL, for instance, is an adiabatic for initial dry steam. Steam thus expanding fogs up with particles of condensed steam forming throughout the mass, and a rainstorm begins in the cylinder at each stroke. That steam does its full work due to expansion, and EL falls more rapidly than the hyperbola. But from L to J the steam does but partial work, viz., that under the compression curve MJ. The action LJ and MJ, of combining, is "isodynamic," that is, no external work is performed. Both these curves may therefore be drawn as isodynamic curves, they being very nearly hyperbolic.

If the steam is reheated to superheat in the receiver, the line IN will be the adiabatic for superheated steam, also the line NG. Such lines fall much more rapidly than the hyperbola. Compression lines, as in cushioning, rise more rapidly than the hyperbola, because the steam is superheated by the compression, and if any water is present, there may be re-evaporation.

The equations of all these curves may be expressed approximately by the equation,

$$pv^x = \text{constant}.$$

in which, for the Mariotte law of expansion, $x=1$. To supply the constant we may put the equation in the form

$$\frac{p}{p_1} = \left(\frac{v_1}{v}\right)^x \quad . \quad . \quad . \quad . \quad . \quad (1)$$

An equation which is approved for ordinary purposes by high authority, and will be adopted in thepresent case.

TABLE E:—OF VALUES OF x, AND AUTHORITY.

Values of $x=$	Adiabatic expan. Initial saturation, s. m	Curve of saturation.	Adiabatic for superheated steam. a	Isodynamic for superheated steam. n
Rankine.	$\frac{10}{9}=1.11111$	$\frac{17}{16}$, and 1.0646	1.3	Not given.
Cotterill.	$1.035+\frac{\infty}{10}$	$\frac{17}{16}$, and 1.0646	"	Bet. s and 1
Zeuner...	" "	1.0646	$\frac{4}{3}=1.3333$	1.0456
Röntgen.	" "	"	"	"

This table and the equation will give us the equation of any curve desired.

In these we not only have authority for a variety of values of x, but in m the "steam quantity" ∞ suggests that the value of x may be determined even by estimation from such facts as may effect ∞. Accordingly, for an expansion where only partial work of expansion is performed, x may be found to lie between $\frac{4}{3}$ and 1.0456 for superheated steam. When $\infty=1$ we have $x=1.135$. It is to be observed, however, that the form of curve, according to equation (1), is dependent on the value of the exponent, so that the general equations may be considered as suitable for any forms of these lines.

The table shows a very considerable range in the value of x, some going so far from $x=1$, as to advise the abandonment of the Mariotte law of expansion in which $x=1$. An example of carefully computed volumes for several curves and chosen pressures will serve to illustrate the departure of the expansion lines of different kinds from each other.

TABLE F.—EXAMPLE. STEAM.

Absolute pressures.	Common hyperbola.	Saturation curve.	Adiabatic curve.
lbs.	vols.	vols.	vols.
23.	122.0	122.0	122.0
50	56.1	58.3	60.6
83	33.8	36.5	38.4

Thus for these curves running through a point at 23 lbs., and volume 122, at about a threefold greater pressure the saturation curve deviates 8 per cent. and the adiabatic curve 14 per cent. in volume from the hyperbola.

But as the curve for which $x=1$, viz., the common hyperbola drawn by its asymptotes, is so very convenient for tracing, (see Fig. 10,) and falls so near the required curves, we are lead to look for some rule by which to "correct over" from the hyperbola to the curves sought. Such rule was given by the writer in the *American Muchinist*, for Feb. 10, 1883, the convenience of which assures the desirability of the following abstract here:

Some engineers judge of the steam action, leakage of piston, valves, &c., by a simple reference to the hyperbola. Then what here follows could be dispensed with, but the more fastidious will probably wish to compare with other curves, which can be easily laid off from this hyperbola by correction, or difference quantities as follows, as hinted by Cotterill, page 341.

Differentiate the equation given above, regarding x and v as variable. The result can be reduced to

$$\frac{dv}{v}=\frac{dx}{x}\text{ hyp. log. }\frac{v_1}{v}=\frac{dx}{x}\times k$$

in which v_1 is one volume, as, for instance, *a*, Fig. 16, and *v*, the other, for any point as *c;* and *k*, a coefficient. The logarithmic part, or coefficient, *k*, may be tabulated, and becomes a convenient coefficient, or factor, for the second member of the equation, or for the quantity $\frac{dx}{x}$ which quantity is definitely known as soon as *x* is decided upon. For instance, take $x=\frac{10}{9}$ for the adiabatic curve, it being 1 for the common hyperbola. Then *dx* will be the difference of these values, or $\frac{10}{9}-1=\frac{1}{9}$, and $\frac{dx}{x}=\frac{1}{10}$.

If the coefficient were 1, then the correction *dv* would be a tenth of *v*.

More definitely, suppose in Fig. 16, $OB=v_1$ and $OA=v$; then the correction is $ce=\frac{1}{10}$ OA, for the values just named.

The values for the coefficient, *k*, are given in the table G.

TABLE G:—OF COEFFICIENTS k.

$\frac{v_1}{v}$	k	$\frac{v_1}{v}$	k
1.1	.095	3.3	1.194
1.2	.182	3.4	1.224
1.3	.262	3.5	1.253
1.4	.336	3.6	1.281
1.5	.405	3.7	1.308
1.6	.470	3.8	1.335
1.7	.531	3.9	1.361
1.8	.588	4.0	1.386
1.9	.642	4.2	1.435
2.0	.693	4.4	1.482
2.1	.742	4.6	1.526
2.2	.788	4.8	1.569
2.3	.833	5.0	1.609
2.4	.875	5.5	1.705
2.5	.916	6.0	1.792
2.6	.955	6.5	1.872
2.7	.993	7.0	1.946
2.8	1.030	7.5	2.015
2.9	1.065	8.0	2.079
3.0	1.099	8.5	2.140
3.1	1.131	9.0	2.197
3.2	1.163	9.5	2.251
		10.0	2.303

TABLE H:—OF VALUES OF $\frac{dx}{x}$.

For the saturation curve,

$$x=\frac{17}{16}, \text{ and } \frac{dx}{x}=\frac{1}{17}=.0588.$$

For the adiabatic curve,

$$x=\frac{10}{9}, \text{ and } \frac{dx}{x}=\frac{1}{10}=0.1.$$

For the adiabatic curve, $x=1.035-.1\,\infty$ and

$$\frac{dx}{x}=\frac{.035-.1\,\infty}{1.035-.1\,\infty}.$$

For the isodynamic curve, $x=1.0456$ and

$$\frac{dx}{x}=\frac{.0456}{1.0456}=.0436.$$

For the adiabatic of steam gas, $x=1.3$, and

$$\frac{dx}{x}=\frac{.3}{1.3}=.2308.$$

For the adiabatic of gases, $x=1.408$, and

$$\frac{dx}{x}=\frac{.408}{1.408}=.29.$$

To illustrate the use of these tables, suppose *ac*, Fig. 16, to be the hyperbola of reference (the common hyperbola),

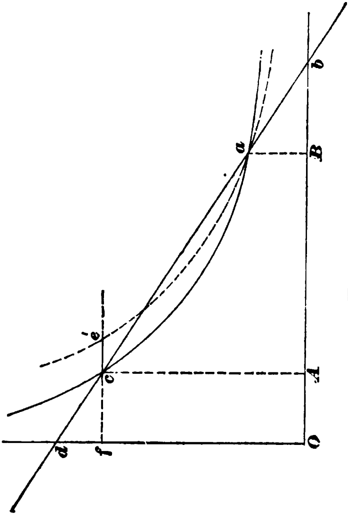

Fig. 16.

with B*a*=23 lbs., A*c*=83 lbs., in absolute pressures; and OB, the volume 122, as in Table F. Then the volume OA, will be 33.8 for the common hyperbola.

Hence for $v_1 = 122$, and $v = 33.8$; $\frac{v_1}{v} = 3.61$.

In Table G, we look under $\frac{v_1}{v}$ for the value 3.61 and find the nearest to be 3.6, but 3.61 is one-tenth the way over toward 3.7. Consequently we look for that value of k, which lies one-tenth the way from 1.281 toward 1.308, and obtain $1.284 = k$. For k, this answers for all the curves.

Now in Table H, we find, for adiabatic expansion of steam,

$$x = \frac{10}{9} \text{ and } \frac{dx}{x} = 0.1$$

consequently

$$\frac{dx}{x} k = 0.1 \times 1.284 = .1284 = \frac{dv}{v}.$$

The correction is therefore $dv = v \times .1284 = 33.9 \times .1284 = 4.9 = ce$, Fig. 16. This added to the volume $v = OA$ gives

$$v + dv = 33.8 + 4.8 = 38.7 = fe,$$

a value which differs less than 1 per cent. from the computed value 38.4 in Table F.

As a second case, suppose the expansion be according to the saturation curve, when by Table H

$$\frac{dx}{x}=\frac{1}{17},$$

then

$$\frac{dx}{x}k=\frac{1}{17}\times 1.284=.0745$$

and $dv=v\times .0745 = 33.8\times .0745 = 2.86$. This added to the volume v gives

$$v+dv=33.8+2.86=36.7$$

a value which differs but slightly from the calculated value 36.5 at the foot of the second column of Table F.

In these calculations we observe that the volume v, with which to multiply, and to which to add, is the one which is to be corrected, viz., in this case we seek the correction $c\ e$, Fig. 16, and hence the volume v is to be taken as AO.

But the rule may be applied the other way. That is to say, we may make c the point of intersection of the two curves, and correct the volume BO. In that case k is to be found as before, and given the contrary sign. In fact the whole quantity $\frac{dx}{x}k$ will be the same as before, except for the contrary sign; if the volumes on the common hyperbola are the same, viz., 33.8 and 122. But in this calculation we should multiply by the volume BO, because the correction is for this volume. That is, the correction is $122 \times .1284 = 15.7$ for the adiabatic expansion, a quantity which, by the accurate method, is found to be 16.6, differing by less than one per cent. of the value of BO to be corrected. For the saturation curve the correction is $122 \times .0745$. &c.

For points beyond *a* the correction is subtractive instead of additive, and multiply tabular quantity by the greater instead the lesser volume; that is, by the volume to be corrected.

Let us apply the first rule to correct the hyperbolic volume 56.1, to the adiabatic volume 60.6, for pressure 50 lbs, absolute; the final volume and pressure being 122 and 23 lbs. respectively.

Dividing greater by lesser volume we get $\frac{152}{56.1}=2.175$. For this ratio of volumes we find k from Table G to be $k=.777$. For the present adiabatic expansion, Table H calls for $\frac{1}{10}$ giving .0777. Multiplying by the volume 56.1 gives the correction of 4.47. Adding this to the volume 56.1 to be corrected, we obtain $56.1+4.47-60.6$; or, in this case, exactly the value given by the logarithmic calculation of Table F.

This method of corrections by which the curves desired can be easily obtained from the simple hyperbola, will greatly facilitate the graphical method of accurately calculating the relative efficiencies or the relative areas of diagrams, or of work performed in the cylinders of compound engines.

ACTUAL EFFICIENCY.

The actual efficiency of the engine is obtained by dividing the whole ft. lbs. of work performed in both cylinders per stroke by the dynamical value of th heat required to produce the steam consumed for that stroke.

Thus, in Fig. 14 the work performed per stroke in ft. lbs. is obtained by multiplying the mean pressure lbs. per square inch in a cylinder by the piston area in square inches, and by the length of stroke in feet, and adding the products thus obtained for all the cylinders. Call this quantity U. The mean pressure is obtained by taking the mean height of the diagrams in the usual way, the scale of heights being that of pressures per square inch. To find the mean height of a diagram by aid of the planimeter, find the area by the instrument and divide that area by the length of the diagram expressed to the proper unit. If the instrument gives the area in square inches divide the area by the length in inches, and lay off the result as a height in inches on the diagram. A horizontal line drawn through this point is the line of mean pressure, and that pressure is to be measured by the scale of pressures.

The steam consumed per stroke has already been stated to be Ea, Fig. 14, which must now be measured with the proper scale. Measured by the same scale which makes BC the volume of the high cylinder in cubic feet, gives the quantity of steam Ea in cubic feet, as desired. Call this V'_1.

The heat required to produce this steam used at a stroke of the high-pressure cylinder will be the total heat of evaporation from the temperature of feed water, and at the temperature of boiler pressure.

Let V_1' = the volume of steam required per stroke of high cylinder in cubic feet, = Ea, Fig. 14.

D_1 = the weight per cubic foot of the steam Ea, Fig. 14, as supplied to the engine.

U = the ft. lbs. of work performed per stroke of whole engine.

H_1 = the dynamical value of the total heat of evaporation per pound from 32° F. at the temperature of boiler pressure steam; that is, the heat to raise the temperature of the water from 32° F. to the temperature of the steam in boiler, added to the latent heat of evaporation at the latter temperature. (Convenient values are given for this in the tables of *Rankine's Steam Engine.*)

t_1 = the actual temperature of the feed water.

J = Joule's equivalent, = 772 ft.-lbs.

Then the total heat consumed per stroke of engine, or of steam E*a*, Fig. 14, in dynamical value, or ft.-lb. units, is

$$H = V_1' D_1 \Big(H_1 - J(t_1 - 32°)\Big) \quad . \quad (2)$$

$$\text{and the efficiency} = \frac{U}{H} \quad . \; . \; . \; . \; . \quad (3)$$

Area of Diagrams by Calculation.

Some of the more simple diagrams may be more readily calculated than drawn on a drawing-board and measured.

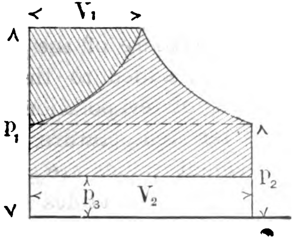

Fig. 17.

For such a diagram as shown in Fig. 17 the area of that portion falling under the expansion line between two verticals and above the vacuum line for one pound of steam in the usual notation is

$$\int pdv \quad . \quad . \quad . \quad . \quad . \quad . \quad (a)$$

where p_1 as shown, is the specific pressure of the steam, or absolute pressure per unit surface; usually pounds per square foot; v_1 = the specific volume, or volume per unit weight; usually cubic feet per pound; and where p is the varying pressure under the curve.

This is for one pound, and not for the actual volumes V_1 and V_2. Hence the area in (a) is simply *like* that designated in Fig. 17, but not equal to it.

But in the present case it will probably be most convenient to apply our reasoning to the actual volumes V_1, V_2, and V' of the high cylinder, low cylinder and receiver respectively.

For the present convenience let $A =$ the area of the high-pressure piston, and L the length of its stroke. Then

$$V_1 = AL,$$

or multiplying through by P_1

$$P_1V_1 = P_1AL = \text{actual area under the admission line} \quad . \quad . \quad . \quad . \quad (4)$$

Similarly below the expansion line

$$\text{actual area} = \int_{L_1}^{L_2} PA\,dL \quad . \quad . \quad . \quad (5)$$

if $L_1 =$ length of the admission line, and $L_2 =$ length of the exhaust line. The pressure P is here variable, but according to (1), the zero of volumes being noted, is

$$\frac{P}{P_1} = \left(\frac{L_1}{L}\right)^x = \left(\frac{V_1}{V}\right)^x$$

Hence (5) becomes

$$\text{actual area} = AP_1L_1{}^x \int_{L_1}^{L_2} \frac{dL}{L^x}$$

$$= AP_1L_1{}^x \frac{1}{1-x}\left(L_2^{\,1-x} - L_1^{\,1-x}\right);$$

or, since $L_2 = L_1 + l$, where l is the length under the expansion line.

Actual area

$$= AP_1L_1{}^x \frac{1}{1-x}\left\{(L_1+l)^{1-x} - L_1^{\,1-x}\right\}$$

$$= AP_1L_1 \frac{1}{x-1}\left\{1 - \left(\frac{L_1}{L_1+l}\right)^{x-1}\right\}$$

$$= \frac{P_1V_1}{x-1}\left\{1 - \left(\frac{V_1}{V_2}\right)^{x-1}\right\}$$

$$= P_1V_1 \frac{1 - \frac{1}{r^{x-1}}}{x-1} \quad . \quad . \quad . \quad (6)$$

that is, the area under any expansion or compression line is equal the rectangle to the pressure and volume of the highest point in the curve multiplied by the function of the ratio r, of expansion, and of the exponent x as shown.

To express the area in terms of the rectangle to the pressure and volume of the lowest part of the expansion curve we have, according to eq. (1).

$$\frac{P_1}{P_2}=\left(\frac{V_2}{V_1}\right)^x=r^x$$

and $$\frac{V_2}{V_1}=r$$

$$\therefore \qquad P_1V_1=P_2V_2r^{x-1}$$

and hence we may write in place of (6)

$$\text{actual area}=P_2V_2\frac{r^{x-1}-1}{x-1} \quad . \quad . \quad (7)$$

For the particular case $x=1$ the equation becomes inapplicable. But for this case the general integral gives the

$$\text{Actual area}=P_1V_1 \text{ hyp. log. } r \quad . \quad (8)$$

which is equal to the rectangle to the highest point of the curve, multiplied by the hyperbolic logarithm of the rate of expansion. These logarithms are found in Table B.

As x is here $=1$, we have

$$P_1V_1=P_2V_2$$

and hence we may put in place of (8) when more convenient,

Actual area$=P_2V_2$ hyp. log. r . (9)

Applying these to Fig. 17, observing that it may be taken for the diagram of a compound engine in which there is no clearance nor receiver, and no cut-off to either cylinder, the cylinder volumes being V_1 and V_2, we have to add to the expansion area of (6) to the rectangle P_1 V_1 and deduct the rectangle below the exhaust line P_3V_2; and hence, calling the work performed by this engine per stroke U, we have, by application of (6),

$$U=P_1V_1+P_1V_1\frac{1-\frac{1}{r^{m-1}}}{m-1}-P_3V_2$$

$$=\frac{P_1V_1}{m-1}\left(m-\frac{1}{r^{m-1}}\right)-P_3V_2 \quad . \quad . \quad (10)$$

P_3 being the back pressure.

This is the same expression as obtained for a single cylinder engine, the whole expansion taking place in the one cylinder.

The exponent x is probably m in this instance for the reason that there is no isodynamic "drop" in the pressure between the cylinders.

Similarly for the case $x=1$ we have, by (8),

$$U=P_1V_1(1+\text{hyp. log. } r)-P_3V_2 \qquad (11)$$

I. Case of Fig. 11.

Calling the volume of admission BC$=V_1'$, and the ratio of expansion in the high-pressure cylinder r_1, we have

$$\frac{V_1}{V_1'}=r_1$$

V_1 being the volume of the high cylinder, and to be so regarded in every application. Hence V_1' in the present case is to replace V_1 of Eq. (10) or (11). Our expression for the whole work of engine per stroke is, therefore,

$$U=\frac{P_1V_1'}{r_1(m-1)}\left(m-\frac{1}{r^{m-1}}\right)-P_3V_2 \quad . \quad . \quad (12)$$

r_1 being the ratio of expansion in the high-pressure cylinder itself, and r the ratio of the entire expansion C to G, also P_1=the boiler pressure (absolute), V_1 the volume AE of high-pressure, and V_2 the volume AF of the low-pressure cylinder.

The work performed in the low-pressure cylinder alone, according to (6), is

$$\mathrm{AHDGFA} - P_3V_2 = \mathrm{EDGFE}\frac{\mathrm{AF}}{\mathrm{EF}} - P_3V_2$$

$$= \frac{\mathrm{AE}\times\mathrm{ED}}{m-1}\left\{1 - \frac{1}{\left(\frac{\mathrm{AF}}{\mathrm{AE}}\right)^{m-1}}\right\}\frac{\mathrm{AF}}{\mathrm{EF}} - P_3V_2$$

$$= \frac{P_1V_1}{r_1{}^m(m-1)}\left\{1 - \left(\frac{V_1}{V_2}\right)^{m-1}\right\}\frac{V_2}{V_2 - V_1} - P_3V_2$$

The work performed in the high-pressure cylinder is, of course, the difference between the last equation and (12).

For the case $x=1$, we have

$$U = \frac{P_1V_1}{r_1}(1 + \text{hyp. log. } r) - P_3V_2$$

and the work performed in the low-pressure cylinder according to (8) is

$$= \mathrm{AE}\times\mathrm{ED}\frac{\mathrm{AF}}{\mathrm{EF}} \text{ hyp. log. } \frac{\mathrm{AF}}{\mathrm{AE}}$$

$$= \frac{V_1P_1}{r_1}\ \frac{V_2}{V_2 - V_1} \text{ hyp. log. } \frac{V_2}{V_1}$$

II. CASE OF FIG. 12.

As to the exponents x for this case, the combined expansion and compression FJ and HJ is isodynamic, that is, unaccompanied with external work, and the steam is probably slightly superheated, so that for the fall from L to I the proper value for x is probably n, or nearly it. Granting this, then the x for the other expansion should be a, or between a and m, as may be judged by the engineer in particular cases. With these values of x, we have

$$\frac{P_i}{P_n}=\left(\frac{V_2+V'}{V_1+V'}\right)^a$$

$$\frac{P_1}{P_i}=\left(\frac{IJ}{V_1}\right)^n$$

$$\frac{P_n}{P_i}=\left(\frac{IJ}{V_2}\right)^a$$

whence

$$\frac{CL}{CI}=\frac{P_1}{P_i}=\left(\frac{V_2}{V_1}\cdot\frac{V_1+V'}{V_2+V'}\right)^n$$

$$\frac{CL}{DN}=\frac{P_1}{P_n}=\left(\frac{V_2}{V_1}\right)^n\left(\frac{V_2+V'}{V_1+V'}\right)^{a-n}$$

by aid of which the diagram can be accurately constructed. The subscripts here refer to the Figure.

For an infinite receiver $V'=\infty$; and the equations for pressures reduce to

$$\frac{P_1}{P_i}=\frac{P_1}{P_n}=\left(\frac{V_2}{V_1}\right)^n. \quad . \quad . \quad (13)$$

By application of Eq. (7), observing that for IN the ratio of expansion is

$$=\frac{V_2+V'}{V_1+V'},$$

we have,

$U=$

$$P_1V_1+P_n\frac{V_2+V'}{a-1}\left(\left(\frac{V_2+V'}{V_1+V'}\right)^{a-1}-1\right)-P_3V_2$$

and eliminating P_n, $U=P_1V_1\times$

$$\left\{\begin{array}{l}1-P_3V_2\\ +\left(\frac{V_1}{V_2}\right)^n\frac{V_2+V'}{V_1(a-1)}\left(\left(\frac{V_2+V'}{V_1+V'}\right)^{a-1}-1\right)\end{array}\right\} (14)$$

If $V'=0$ the receiver is dispensed with, and this case reduces to that of Case I. The above then for $n=a$ reduces to

$$U_1=P_1V_1\left\{\frac{a}{a-1}-\frac{1}{a-1}\left(\frac{V_1}{V_2}\right)^{a-1}-\frac{P_3V_2}{P_1V_1}\right\} (15)$$

which is the same as (10) if a is changed to m, as it should be for the present supposition where $V'=0$ of no isodynamic expansion nor consequent superheating. The change in diagram, Fig. 12, due to making $V'=0$, consists in the raising of the point I up to L while N remains fixed. The diagram then agrees with Fig. 17.

When $x=1$ we have for Fig. 12 and Equation (9)

$$\text{area CIND}=P_n(V_2+V')\text{hyp. log.}\frac{V_2+V'}{V_1+V'} \quad . \quad . \quad . \quad . \quad . \quad (16)$$

Adding BCLF and deducting for back pressure we have for the work performed per stroke of the engine

$$U=$$

$$P_1V_1+P_n(V_2+V')\text{hyp. log.}\frac{V_2+V'}{V_1+V'}-P_3V_2$$

$$=P_1V_1\left\{1+\left(1+\frac{V'}{V_2}\right)\text{hyp. log.}\frac{V_2+V'}{V_1+V'}\right\}$$
$$-P_3V_2 \quad . \quad . \quad . \quad . \quad (17)$$

It is to be observed that the zero of volumes for the expansion IN is at A, and for the curve LN we have, for $x=1$,

$$P_nV_2=P_1V_1$$

If $V'=0$ we have Eq. (11), as we evidently should.

IIA. CASE OF CUT-OFF AND EXPANSION IN THE HIGH-PRESSURE CYLINDER, TOGETHER WITH A RECEIVER.

Fig. 18 is the diagram representing the present case, where BC is the volume of the high-pressure cylinder, BD the volume of the low-pressure cylinder, and AB the volume of the receiver.

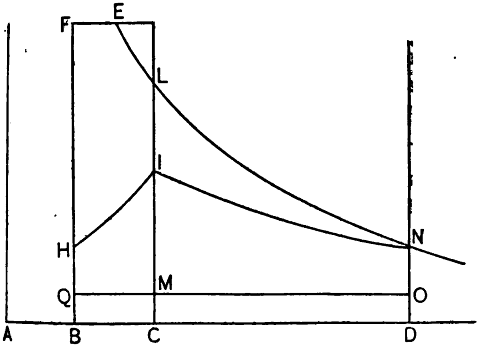

FIG. 18.

The admission at boiler pressure P_1 is from F to the cut-off at E. From E to L is the expansion curve answering to the expansion in the high-pressure cylinder, L to I is the fall of pressure due to exhaust from this cylinder into the receiver AB, and IN is the expansion curve for the forward stroke of the low-pressure cylinder, the zero of volumes for which curve is at A, because at the beginning of the stroke the volume AC is that which fills the receiver and high-pressure cylinder, while at the end of the stroke the same steam has the volume AD of the receiver and low-pressure cylinder.

The work done in the high-pressure cylinder per stroke is represented by the area HFELIH, while the same for the low-pressure cylinder is QHINOQ, OQ being the exhaust line.

Let r_1=ratio of expansion in the high-pressure cylinder, giving

$$r_1 = \frac{BC}{EF}.$$

An expression for the

$$\text{area BCLEFB} = \frac{\mathrm{BF} \times \mathrm{FE}}{m-1}\left(m - \frac{1}{r^{m-1}}\right)$$

$$= \frac{P_1 V_1}{r_1(m-1)}\left(m - \frac{1}{r^{m-1}}\right) \quad . \quad . \quad (18)$$

is given in (12), omitting exhaust area.

Referring to the equation preceding (14), we see that it may be modified to represent the work per stroke in the present case by substituting (18) for the first term, and by substituting $P_1 \div r_1{}^m$ for "P_1" of the second term, because there "P_1" = the terminal pressure CL, Fig 18, while BF must stand for P_1. Making these changes in (14) we have, for the work per stroke of both cylinders for the case now considered,

$$U = \frac{P_1 V_1}{r_1(m-1)}\left(m - \frac{1}{r^{m-1}}\right) +$$

$$\frac{P_1 V_1}{r_1{}^m}\left(\frac{V_1}{V_2}\right)^n \frac{V_2 + V'}{V_1(a-1)}\left(\left(\frac{V_2 + V'}{V_1 + V'}\right)^{a-1} - 1\right)$$

$$-P_3 V_2 \quad . \quad . \quad . \quad . \quad . \quad (19)$$

$$=P_1V_1\left\{\frac{m-\frac{1}{r^{m-1}}}{m-1}\right.$$

$$+\frac{V_2+V'}{V_1r_1{}^m(a-1)}\left(\frac{V_1}{V_2}\right)^n\left(\left(\frac{V_2+V'}{V_1+V'}\right)^{a-1}-1\right)$$

$$\left.-\frac{P_2V_2}{P_1V_1}\right\} \quad . \quad . \quad . \quad . \quad (20)$$

The work performed in the low-pressure cylinder is found by taking the last two terms of (19) for the area MINO, multiplying it by BD, and dividing by CD, that is,

$$\text{area QHINOQ}=\text{MINO}\times\frac{V_2}{V_2-V_1} \quad . \quad (21)$$

For the case $x=1$ we have the 1st term of (11) as an expression for the area of the diagram of the form BCLEF, Fig. 18, where the "V_1" in (11) is $=$FE. But here BC$=V_1=r_1$FE$=$"V_1"r_1, whence

$$\text{"}V_1\text{"}=\frac{V_1}{r_1}.$$

Putting this value of "V_1" into the first term of (11), and we have

$$\text{BCLEFB}=\frac{P_1V_1}{r}(1\times\text{hyp. log. }r_1) \,.\, (22)$$

The area for CIND is

$$AD \times DN \text{ hyp. log. } \frac{AD}{AC}$$

and $BD \times DN = BC \times CL = V_1 \times CL.$

But $r_1 \times CL = P_1 = BF,$

and $DN = \frac{P_1 V_1}{r_1 V_2}.$

Hence the area

$$CIND = \frac{P_1 V_1}{r_1 V_2}(V_2 + V') \text{ hyp.log.} \frac{V_2 + V'}{V_1 + V'} \quad (23)$$

and the area representing the work done by both cylinders per stroke is the sum of (22) and (23), less P_3V_2.

The work per stroke of the low-pressure cylinder is

$$= (23) \times \frac{BD}{CD} - P_3 V_2 \quad . \quad . \quad . \quad (24)$$

and this taken from the work performed by both cylinders per stroke gives the work performed per stroke of the high-pressure cylinder.

The expression (24) becomes

$$\frac{P_1 V_1}{r_1}\left(1 - \frac{V'}{V_2}\right)\left(\frac{V_2}{V_2 - V_1}\right) \text{hyp.log.} \frac{V_2 + V'}{V_1 + V'} - P_3 V_2$$

in which, if we neglect the last term and make $V'=0$ we obtain the

Case of no Receiver,

$$\frac{P_1V_1}{r_1}\frac{V_2}{V_2-V_1} \text{ hyp. log. } \frac{V_2}{V_1}$$

If we put the ratio of expansion $\frac{V_2}{V_1}$ in the low pressure cylinder$=r_2$ and observe that CL is then the initial pressure in the low-pressure cylinder, which for the moment we may call $P'=\frac{P_1}{r_1}$ the above becomes

$$\frac{P'V_2}{r_2-1} \text{ hyp. log. } r_2=V_2p'm \quad . \quad . \quad (25)$$

an expression which is essentially the same as that given just preceding Table C.

Similarly for $V'=0$ and omitting P_3V_2, the work for the high-pressure cylinder per stroke is

$$\frac{P_1V_1}{r_1}\left(1+\text{hyp.log.}r_1-\frac{1}{r_2-1}\text{hyp.log.}r_2\right) (26)$$

and for both cylinders together the work per stroke is

$$\frac{P_1V_1}{r_1}\left(1 + \text{hyp. log. } r_1 + \text{hyp. log. } r_2\right). \quad (27)$$

By placing the expressions for the work performed in the two cylinders equal to each other we obtain, after reduction, .

$$\text{hyp. log. } r_1 = \frac{r_2+1}{r_2-1} \text{ hyp. log. } r_2 - 1 \;.\;.\quad (28)$$

an expression which gives the value of r_1 the ratio of expansion in the high-pressure cylinder in terms of r_2 the ratio of the volumes of the cylinders and also the ratio of expansion in the low-pressure cylinder.

Example: CASE OF PART I.

Placing $r_2=3.464$
we get $r_1=3.464$

for the only case where the work in one cylinder can equal that in the other, and yet with equal ratios of expansion. But this equality of work fails when there is an appreciable exhaust pressure.

As a 2d example take $r_2=2$

Then for equal work performed in the cylinders we have $r_1=2.94$

Indeed, equation (28) shows that there are an infinite number of values of r_2 which will satisfy r_1 for equal work performed in the cylinders.

Effect of a Receiver.

As the volume of the receiver is changed from zero to infinity, the point I, Fig. 12, falls from L to the horizontal through N; while the point N remains stationary. To find the point I on a drawing-board for any set of values of V_1 V_2 and V′; first find N by aid of (13) then draw the adiabatic NI with the proper exponent, and with A for the zero of volumes.

We thus arrive at the important conclusion that by increasing the size of the receiver we diminish the amount of work performed by the same weight of steam, and to the extent due to the corresponding lowering of the line NI.

In the volume V′ of receiver must be included all the space about the valves and of the communicating pipes between the cylinders, as well as that in a receiver proper. Thus the superheating chambers sometimes employed between the cylinders must be counted in.

The effect of superheating in the receiver is that to change the pressures and not the volumes. Such superheating simply raises the lines NI and HI. If the steam is dry without a superheater, then with it these lines will be raised nearly in proportion to the elevation of the absolute temperature by superheating.

COMPOUND PUMPING ENGINES.

In rotative compound pumping engines any of the foregoing cases will apply as well as for other purposes. But for non-rotative pumping engines some difficulty is experienced in maintaining motion near the end of the stroke, due to the fall of pressure of the expanding steam. To admit a puff of steam near the end of stroke for a last lift that shall carry the engine over is very wasteful of steam. To illustrate by a diagram, suppose the expansion be carried from F to H, Fig. 19, at which latter point the engine fal-

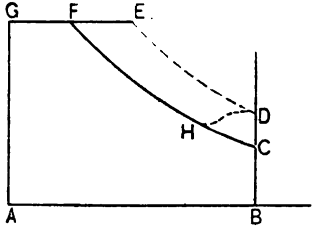

Fig. 19.

ters. Now to admit a portion of steam that shall cause the pressure to rise to D, and add the area HDC over what it would have been without the added steam; the effect is a loss of work from what it would have been had the same steam been admitted at the beginning of the stroke as represented by the area HFEDH, and the expenditure of steam is increased by the ratio EG÷FG. (This estimate, however, will be slightly modified by the fact that the steam thus admitted is reduced in pressure at the admission and superheated somewhat.)

Owing to this difficulty of faltering in non-rotative engines it is customary to use no cut-offs on the cylinders, but to secure the expansion by passing the steam from the high-pressure into the low-pressure cylinder.

If there be no receiver the diagram is of the kind shown in Fig. 17, and the work performed per stroke is given by equation (12) for $r_1 = 1$. An example of such an engine is shown in Fig. 9, though the intermediate steam pipe must have the effect of a receiver in small degree.

If there be a receiver and no cut-offs, the diagram is shown in Fig. 12. In practice the communicating passage between the cylinders amounts to a small receiver in effect, however small it may be, and the expression of amount of work per stroke is given in (14). The receiver has the effect to reduce the work per stroke by the ratio, see Fig. 12,

$$\frac{QFLINOQ}{QFLKNOQ} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

if there be no reheating in the receiver. But in reheating great advantages may arise, as already considered.

THE TANK ENGINE.*

An interesting form of compound engine for pumping has been introduced to some extent by H. R. Worthington, and by Cope and Maxwell, in which there are two cylinders, one larger than the other, and an intermediate "tank" or receiver.

The high-pressure cylinder in this engine is supposed to take its steam at constant pressure from a boiler during its full stroke, then to exhaust into the receiver. The back pressure is the pressure of the steam in the receiver, which will be somewhat variable according to relation of sizes of receiver and cylinders. The low-pressure cylinder steam supply will also vary somewhat in pressure according to pressure in the receiver. This cylinder exhausts into the air, or into a condenser; the back pressure due to which is assumed constant in the calculations.

As to the piston movements three are possible, viz., 1st, they may have entire independence; 2d, they may be limited to an equal number of strokes per minute, with initial points in certain relation; and, 3d, they may be further limited by periods of tarrying. The latter is the case with the engine now considered.

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* Extracted from a paper read before the American Society of Mechanical Engineers.

As to the valve movements, four are possible, each differing from another in efficiency of the engine. Two are selected as special objects for investigation here, viz., the one with the lowest efficiency, and that with the highest. But all these efficiencies approach, and, finally, have a common value as the receiver is enlarged to infinity. For this latter and special case the formulas are much simplified.

One important fact to be pointed out is that with a certain mode of working the valves, a comparatively small receiver gives a higher efficiency of steam than a larger, or infinite one.

The engine, with two pistons and one receiver, is briefly shown by diagram in Fig. 18, where *b*, *d*, are admission valves for high steam, into small cylinder; *e*, *f*,

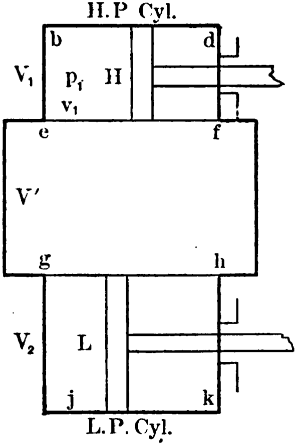

FIG. 20.

exhaust valves from the small cylinder into the receiver; *g*, *h*, admission valves from the receiver to the low-pressure cylinder; and *j*, *k*, exhaust valves from the low cylinder.

The action of the pistons, except where mentioned as otherwise, is considered as alternating; that is to say, one piston stands still at the end of the cylinder, while the other makes its stroke, and conversely. This is the third above mentioned.

The diagram will enable us to indicate the fact of different ways for working the valves. For instance: 1st, suppose *b* and *f* open, *e* and *d* closed, piston H moving, and piston L standing at end of cylinder. Then with *j* and *k* closed, *g* and *h* may be either opened or closed. If closed, the steam from *f* accumulates only in the receiver, while if open, it flows into both the receiver and L cylinder. This option in the valves *g* and *h* occasions two valve movements. Again: 2d, suppose piston L moving, while piston H stands at the end of the stroke, then with *b* and *d* closed, *e* and *f* may be either opened or closed, a second option which occasions two separate valve movements, for this side of the engine. In the combined action of both sides, it appears from the fact that these options are independent of each other, there results four possible valve movements, two of which, as above mentioned, are selected for investigation and comparison, and a shown in Fig. 21. The 1st is the leas and the 2d the most efficient of the four.

In the 1st, and also 2d, of Fig 21, the engine is represented in the four relations necessarily undergone for the completing a cycle of operations. For instance in 1st, we have piston H moving while piston L stands, then H stands while L moves, next H moves opposite to the first as L stands, and finally H stands as L moves, contrariwise to that of its first stroke. The next move is the same as the first, and thus they continue in repetition. Piston-rods are shown by arrows, which indicate direction of motion. Broken arrows mean standing still.

In Fig. 21 all open valves are indicated by arrows. Feathered arrows are put for valves necessarily open, while naked arrows are at valves in option. In the light of these remarks, the valve movements of 1st and 2d may be traced throughout, also steam action. For convenience let the four relations of Fig.

21 be designated as A, B, C, and D, as shown.

Then in 1st: for the A relation we have H moving with high steam on one side and exhaust on the other. These are necessary conditions while H moves, and hence the arrows are feathered. The exhaust, however, is shown as into the receiver not only, but the L cylinder as well, because the valve to the L cylinder is open. But that valve might be closed when the steam would be confined to the receiver with a more rapid rise of pressure. The latter condition is shown in the A relation of 2d, hence this arrow is left naked.

Again in the B relation, the H piston is standing while L is making its stroke. The valve between H and the receiver is closed in 1st, or it may be open as in 2d, and hence the arrow is featherless, as shown.

These references suffice to explain the C and D relations also.

V. SOLUTION FOR THE FIRST VALVE MOVEMENT OF FIG. 21.

Before making statements of steam action it will be necessary to know specifically the condition of the steam at every point as it passes through the engine. For the 1st part of Fig. 21, the complete diagram of steam action, for continuity of engine movement, is given in Fig. 22.

Let V_1=volume of the high pressure cylinder, V_2=volume of the low-pressure cylinder, and V'=volume of the receiver. These volumes are shown on the diagram in their proper relations. Also the absolute pressures P_1 of the boiler, P_2 of terminal expansion, and P_3 of the exhaust, are shown. Other pressures will be indicated according to lettering of diagram, as, for instance, $P_i = P_g$ being the pressures at the points I and G. In short, the pressures and volumes are represented by vertical and horizontal distances respectively, with A for the zero or origin.

The Complete Diagram and Indicator

Cards.—Now, referring to 1st of Fig. 21, we see that in the A relation the low-pressure or L-piston is standing at the end of its stroke, and with the valve between that cylinder and the receiver open; the steam that made the last L-stroke being retained. Hence the L-cylinder and receiver both together are serving as receiver, while the high-pressure or H-piston is making its stroke and forcing the steam from the H-cylinder into the receiver. The H-piston is moved by steam from the boiler, the same being admitted at full pressure P_1, and full stroke.

Now in Fig. 22, FL represents this high-pressure line of P_1, while KG represents the compression line of the H-piston stroke. It is to be observed that at the initial return of H, the volume is $AC = V_1 + V_2 + V'$; and at the end, it is $AD = V_2 + V'$.

Next, in the B-relation of 1st of Fig. 21, the valves of L are reversed, as H completes its stroke, the steam in the L-cylinder being exhausted. In Fig. 22

this exhaust is shown by a cutting off of the steam GI, and leaving only that in the receiver, of volume $V'=AE$, to act upon the L-piston.

The H-piston now stands while the L-piston makes its stroke with a change of active steam volume from V' to $V'+V_2$; and producing the expansion line IH, Fig. 22. Next the valves of L, Fig. 21, remain, while the valves of H reverse; so that the L-piston stands while the H-piston moves as in the C movement.

Now as the valves of H reverse, the high steam exhausts into the receiver and L-cylinder together, giving the expansion line FJ for the high steam, and the compression line HJ for the low steam, till both arrive at a common pressure at J. The expansion line FJ is to be drawn with C as the point of zero volumes and pressures; while HJ has A for the like point. Here the steam, isodynamically brought down from P_1 at F, will be hotter than that brought up from P_2 at H; and as they commingle, the volume JK decreases by giving up heat, and the volume AB increases by receiving that heat. But as the same weight of steam must necessarily be exhausted from the high cylinder as from the low at a stroke, it follows that the steam JK introduced must have equal weight with the steam GI exhausted. Hence the volume JK must shrink by cooling to some volume J′K, such that J′ lies on the expansion curve IH; because when AC is compressed along KG to AD, then J′K must compress to GI, and AB′ to AE. The curves KG and J′I are of like sort, with the point A for the zero of pressures and volumes.

Now H commences its stroke with a back pressure $P_j = CK$, and the volume $AC = V_1 + V_2 + V'$. At the end of the stroke, this volume has been reduced to AD, giving the compression line KG. At the end of this stroke the valves of L are reversed, causing the immediate exhaust from L of the volume $GI = V_2$, when in the L-stroke the expansion curve IH, etc., is formed as before.

Thus it appears that FGKL is the

diagram for the H-cylinder, while HIQS is the diagram for the L-cylinder. The work done by these cylinders will be proportional to these areas. The back pressure is $EQ=P_3$.

Nature of the Expansion and Compression Lines.—Having traced out the action of the engine and of the steam as it is worked in it, we are prepared to inquire into the specific nature of the various lines of the complete diagram, Fig. 22, from the standpoint of thermodynamics. Thus the compression curve KG is adiabatic, and such that if the steam is saturated at K, it is superheated at G by the compression. Hence, as the steam will be shown to be not far from saturated at K, this curve is an adiabatic for superheated steam. The same is true of J′I, because as K is compressed to G, J′ is compressed to I. After the exhaustion of the volume GI, the curve IJ′ is retraced by expansion with the same steam as was just compressed along J′I. But with J′H the case is to some extent different, before explaining which, we must consider JH and JF. With respect to the latter, it is to be observed that at the end of the stroke of the L-piston, the volume $V'+V_2$ is ready to receive the high steam exhaust from the H-cylinder. Here the low-steam pressure is $DH=P_2$, and high-steam pressure $DF=P_1$; while the volumes are $V'+V_2=AD$, and $V_1=DC$, respectively. Now as the valve from H to the receiver opens, these two portions of steam coalesce * into a common body, whose volume is simply the sum of the previous partial volumes; that is to say,

$$(V_2+V')+V_1=V_1+V_2+V';$$

and the resulting pressure is BJ. In this act no exterior work is performed by the mass of steam in the volume AC, because, for the instant during which this commingling occurs, the pistons of the en-

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* This, however, is only partially true, since some of the steam remains in the *H*-cylinder till forced out through the open valve into the receiver during the return stroke. Though the pressure is common, yet the temperature and density of that remaining in the cylinder will probably differ somewhat from that in the receiver.

gine do not move perceptibly. This act is therefore governed by *Hirn's law*; that is to say, because no external work is done during the change, the internal energy of the mass of steam remains constant. The curve representing this action is the so-called *isodynamic curve.* Hence the pressure BJ is to be found at the intersection of two isodynamic lines FJ and HJ; the first representing the isodynamic expansion of the volume V_1, and the second, the compression of the volume $V' + V_2$.

Though these curves serve to determine the point J, or resulting pressure BJ, yet it is to be understood that they do not represent the operations which actually take place in the two portions of steam, because the low steam is in reality compressed adiabatically from H to J′, and the work B′DHJ′ performed. This work is performed by the expansive action of the high steam while actually expanding along some line FJ′ and not FJ. This curve, however, will lie above the adiabatic through F, because in expanding along FJ′ the steam is not doing its full work, the work actually done being only B′DHJ′ instead of an area extending fully up to the expansion line. Hence, J′ instead of J, represents the actual resulting point of pressure.

Now the points J and J′ are expected to be *very nearly* in a horizontal line, but may not be exactly so for several practical reasons. The expansion J′ H will probably be mostly adiabatic below the saturation point, the point of change from superheat to supersaturation lying somewhere on J′H, but not far from J′. Much of the supersaturation moisture due to this expansion may be precipitated while the L-piston tarries at the end of its stroke, so that the compression heat of HJ′ acts at a disadvantage in re-evaporating that moisture. The moisture thus precipitated may accumulate in the receiver as water, and require to be drawn off by a cock. To show that at the end of the stroke IH the steam is necessarily below the saturation point, we observe that KG and J′I are counter actions which offset each other, as already explained, and hence have no influence on the final condition of saturation. Now, referring to the isodynamic lines, the expansion FJ gives rise to superheat, while the compression HJ occasions condensation. These actions nearly compensate each other, because, though the intensity for FJ is greater than for HJ, yet the quantity of steam concerned in JH is greater : assuming that these exactly neutralize, then the saturation point lies very near to J′, and below it if supersaturation moisture is precipitated in the receiver.

It appears, therefore, that the expansion line IH is of two kinds, viz., from I to some point near J′ it is adiabatic for superheated steam, while from the latter point to H it is adiabatic for supersaturated steam.

To account for JJ′ on the Supposition of Heat.—In this case the drawing-board may be preferred. Use temperatures and a diagram, as shown in the lower part of Fig. 22. As the temperatures for the isodynamic lines through J are each nearly constant, let the absolute temperatures τ_1 and τ_2 of the steam at admission and at the end of J′H be laid off downward, as shown. Draw lines to M and N, meeting JMN. Then draw straight lines AMO and CNO, giving the intersection O. Then the line through O parallel to NJ should give the point J′. The diagram ANC depends upon the law of Gay Lussac, relating to volume and temperature, for constant pressures in gases, viz., by symbols

$$\frac{v}{\tau}=\frac{v'}{\tau'}$$

In superheated steam this very nearly holds true. In the diagram the application is AB : BM : : RO : RM, and CB : BN : : RO : RN. The two portions of steam are first supposed reduced to the common pressure P_j, by the isodynamic or nearly constant temperature expansion FJ, and the like compression HJ. The volume BC then has the temperature τ_1 of P_1, and the volume AB the temperature τ_2 of p_2. The change JJ′ is that by which the portion BC shrinks, and of AB expands, while the temperatures τ_1 and τ_2 merge into one temperature, τ = BR.

Repeated trials should be made until JJ′ is a horizontal line from the intersections J of the isodynamics to the adibiatic IH, and with J′K and GI also horizontal.

But as the correction to the volume AB $= V_j$ is usually less than one per cent., it is probable that the discrepancies in results of efficiency or duty of engine due to it will be too small to merit serious consideration.

Hence, in the following general equations the points J and J′ will be regarded as coincident; and the entire expansion line IH will be treated as of one form, or as characterized by one value of the exponent x in Eq. (1).

Area of the Indicator cards; and General Equations,—To find the area of the diagram, we have from equation (6) and Fig. 22, observing that the rectangle of the pressure and volume of the initial point of the curve GK is AD×DG, and that the ratio of expansion is AC÷AD, area GDCK=

$$P_g\frac{V_2+V'}{a-1}\left\{1-\frac{1}{\left(1+\frac{V_1}{V_2+V'}\right)^{a-1}}\right\}$$

$$=P_g V_1 M \quad . \quad . \quad . \quad (30)$$

if

$$M=\frac{V_2+V'}{V_1(a-1)}\left\{1-\frac{1}{\left(1+\frac{V_1}{V_2+V'}\right)^{a-1}}\right\} (31)$$

For the particular case $a = 1$, the integral becomes, for the small cylinder

$$\left.\begin{aligned}(GDCK)_{a=1} &= AP_gL_1\text{hyp.log.}\left(1+\frac{l}{L_1}\right)\\ &= P_g(V_2+V')\text{hyp.log.}\left(1+\frac{V_1}{V_2+V'}\right)\end{aligned}\right\} (32)$$

In these equations P_g is the absolute pressure DG, =EI.

Similarly for the area IEDH, taking the point A, Fig. 22, as the zero of volumes, we have by aid of (6)

$$IEDH=P_g\frac{V'}{a-1}\left\{1-\frac{1}{\left(1+\frac{V_2}{V'}\right)^{a-1}}\right\}$$

$$=P_g V_2 N \quad . \quad . \quad . \quad (33)$$

if

$$N=\frac{V'}{V_2(a-1)}\left\{1-\frac{1}{\left(1+\frac{V_2}{V'}\right)^{a-1}}\right\}\quad . \; . \quad (34)$$

For $a=1$, we have

$$(IEDH)_{a=1}=P_gV' \text{ hyp. log.}\left(1+\frac{V_2}{V'}\right).$$

Now the work done in the high-pressure cylinder, per stroke, is

$$P_1V_1—GDCK.$$

Also in the low-pressure cylinder it is

$$IEDH-P_3V_2.$$

Hence, when the engine is working in continuity under the first valve movement of Fig. 21, and giving the diagram of Fig. 22, then, for one stroke of each piston in regular order of continuity, the same being the half of a complete cycle of operations detailed in Fig. 21, or, for each high-pressure cylinder full of steam, the work done per stroke of both cylinders is $U=(P_1V_1-GDCK)+(IEDH-(P_3 V_2)$

$$U=P_1V_1-P_g\,V_1M-P_3V_2+P_g\,V_2N \quad (35,)$$

all being in known terms except P_g. Fig.

22 shows that this depends on many things, and that it is in no case, in itself, arbitrary, but that, in an actual engine it is self-adjusting according to pressure of admission, dimensions of cylinders and receiver, mode of valve action, and laws of expansion.

To express P_g in convenient terms, we may write from Fig. 22, and by aid of (1) and the following table, regarding J and J′ as coincident,

$$\frac{P_1}{P_j}=\left(\frac{JK}{FL}\right)^n=\left(\frac{JK}{V_1}\right)^n.$$

Also,

$$\frac{P}{P_j}=\frac{P_g}{P_k}=\frac{P_g}{P_j}=\left(\frac{AC}{AD}\right)^a$$

$$=\left(\frac{V_1+V_2+V'}{V_2+V'}\right)^a=\left(1+\frac{V_1}{V_2+V'}\right)^a$$

Again, from the fact that the intercepts on horizontal lines lying between two curves constructed from (1) with one value for *x* and different values of P_1 V_1, are proportional to the abscissas to the intersection points of the horizontal lines with either curve, we may write,

$$\frac{\mathrm{JK}}{\mathrm{GI}}=\frac{\mathrm{JK}}{\mathrm{V}_2}=\frac{\mathrm{AC}}{\mathrm{AD}}=\frac{\mathrm{V}_1+\mathrm{V}_2+\mathrm{V}'}{\mathrm{V}_2+\mathrm{V}'}$$

$$=1+\frac{\mathrm{V}_1}{\mathrm{V}_2+\mathrm{V}'}.$$

Eliminating JK and combining, we get

$$\frac{\mathrm{P}_g}{\mathrm{P}_j}\cdot\frac{\mathrm{P}_j}{\mathrm{P}_1}=\frac{\mathrm{P}_g}{\mathrm{P}_1}$$

$$=\left(1+\frac{\mathrm{V}_1}{\mathrm{V}_2+\mathrm{V}'}\right)^a\cdot\left(\frac{\mathrm{V}_1}{\mathrm{V}_2}\right)^n\cdot\left(1+\frac{\mathrm{V}_1}{\mathrm{V}_2+\mathrm{V}'}\right)^{-n}$$

$$=\left(\frac{\mathrm{V}_1}{\mathrm{V}_2}\right)^n\cdot\left(1+\frac{\mathrm{V}_1}{\mathrm{V}_2+\mathrm{V}'}\right)^{a-n}\quad . \quad . \quad . \quad (36)$$

Eliminating $\frac{\mathrm{P}_g}{\mathrm{P}_1}$ by aid of (36), we find for the work of the half cycle, or for one stroke each of the two cylinders in continuity,

$$\mathrm{U}=\mathrm{P}_1\mathrm{V}_1$$

$$\left\{1+\left(\frac{\mathrm{V}_1}{\mathrm{V}_2}\right)^n\cdot\left(1+\frac{\mathrm{V}_1}{\mathrm{V}_2+\mathrm{V}'}\right)^{a-n}\cdot\left(\frac{\mathrm{V}_2}{\mathrm{V}_1}\mathrm{N}-\mathrm{M}\right)\right.$$

$$\left.-\frac{\mathrm{P}_3\mathrm{V}_2}{\mathrm{P}_1\mathrm{V}_1}\right\}\quad . \quad . \quad . \quad (37)$$

and for the hyperbolic expansion, where a and $n=1$, we have

$$U'=P_1V_1\left\{\begin{array}{l}1-\frac{V_2+V'}{V_2}\text{hyp.log.}\left(1+\frac{V_1}{V_2+V'}\right)\\-\frac{P_3V_2}{P_1V_1}+\frac{V'}{V_2}\text{hyp.log.}\left(1+\frac{V_2}{V'}\right).\end{array}\right\}\quad(38)$$

all quantities being in known terms. If $a=n$, the term $\left(1+\frac{V_1}{V_2+V'}\right)$ drops from (37).

If $V'=\infty$, and a and n as in (37), we get

$$U''=P_1V_1\left\{1-\left(\frac{V_1}{V_2}\right)^n+\left(\frac{V_1}{V_2}\right)^{n-1}-\frac{P_3V_2}{P_1V_1}\right\}\quad\ldots\ldots\quad(39)$$

If $V'=\infty$, and a and $n=1$, we get

$$U'''=P_1V_1\left(2-\frac{V_1}{V_2}-\frac{P_3V_2}{P_1V_1}\right)\quad..\quad(40)$$

In these expressions,

P_1=absolute pressure of admission, lbs. per square foot=144 (p_1'+14.7), for p_1'=lbs. per square inch of boiler pressure.

P_3=absolute back pressure to low-pressure cylinder lbs. per square foot of the atmosphere for exhausting into air, or of the condenser for condensing engines.

V_1=volume of high-pressure cylinder in cubic feet.

V_2=volume of low-pressure cylinder in cubic feet.

V'=volume of receiver and connecting pipes in cubic feet.

M and N being given by equations (31) and (34).

These equations are useful in calculating the amount of work that can be done by an engine, or in calculations for duty.

In practice any one of these equations can be used according to accuracy required, (37) being necessary for the greatest degree of exactness. It is seen, however, by (1) and the accompanying table that a much wider departure from truth results when $a=1$ than when $n=1$. When the steam is very wet the actual value of a will be much nearer unity than for dry steam. In such case some value of m should be used in place of a. Fig. 22, also, will indicate in some degree to what extent, in any practical case, we jeopardize the result by taking V′ infinite in the calculations.

Pressure at Different Points of Steam Action.—It is often desirable to know the pressure at different points of the steam action in the engine. To this end we have (36) for the terminal pressure, p_g, in the high cylinder, and initial pressure in the low cylinder. This shows that if $a = n$ it matters not with p_g whether $V' = \infty$ or not. From this it appears that for $a = n$ the smaller we make V′ the greater is the proportion of work done by the small and less by the large cylinder.

For the initial and final pressures in the low-pressure cylinder we have the relation

$$\frac{P_g}{P_2} = \left(\frac{V_2 + V'}{V'}\right)^a = \left(1 + \frac{V_2}{V'}\right)^a \quad . \quad . \quad (41)$$

combining with (36)

$$\frac{P_2}{P_1} = \left(\frac{V'}{V_2 + V'}\right)^a \cdot \left(\frac{V_1}{V_2}\right)^n \cdot \left(1 + \frac{V_1}{V_2 + V'}\right)^{a-n} . \quad (42)$$

The 4th equation preceding (36), combined with the 2d, gives

$$\frac{P_1}{P_k}=\frac{P_1}{P_j}=\left(\frac{V_2}{V'}\right)^n\left(1+\frac{V_1}{V_2+V'}\right)^n \quad . \quad . \quad (43)$$

These equations show that for $V'=\infty$ we have

$$\frac{P_g \text{ or } P_j \text{ or } P_2}{P_1}=\left(\frac{V_1}{V_2}\right)^n \quad . \quad . \quad (44)$$

which is the constant pressure of the infinite receiver, and toward which this pressure approximates as V' is made large.

Equation (36) shows that if $a=n$, as it will very nearly for wet steam, the pressure at G or I, $=P_g$, will be entirely independent of V' whether infinite or finite. The effect of this on the diagram is that the lines GK, or IH, swing around the points G and I as pivots, becoming flatter as V' is enlarged, or steeper as it is diminished. Hence, as V' is enlarged, the work produced in V_1 decreases, while in V_2 it increases. Considering these variations, a look at the diagram, Fig. 22, shows that the efficiency of this engine increases with V', and is a maximum when V' is infinite.

When $V'=0$, V_2 does no work, and the engine does worse than a non-compound, but equals it when IQSH=GSS′K.

Relative Areas of Indicator Cards.—It is also desirable to know the relation between the work developed in the high-pressure cylinder and the low. From the expressions (35) and (36) given above, we readily find,

$$\frac{\text{Wk. H.-Cyl.}}{\text{Wk. L.-Cyl.}}$$

$$=\frac{1-M\left(\frac{V_1}{V_2}\right)^n\left(1+\frac{V_1}{V_2+V'}\right)^{a-n}}{N\frac{V_2}{V_1}\left(\frac{V_1}{V_2}\right)^n\left(1+\frac{V_1}{V_2+V'}\right)^{a-n}-\frac{P_3V_2}{P_1V_1}}\ .\ (45)$$

For the hyperbolic expansion this ratio is

$$=\frac{1-\frac{V_2+V'}{V_2}\text{hyp. log}\left(1+\frac{V_1}{V_2+V'}\right)}{\frac{V_1}{V_2}\text{hyp. log.}\left(1+\frac{V_2}{V'}\right)-\frac{P_3V_2}{P_1V_1}}\ .\ (46)$$

If $V'=\infty$, and a and n are as in (45)

$$=\frac{1-\left(\frac{V_1}{V_2}\right)^n}{\left(\frac{V_1}{V_2}\right)^{n-1}-\frac{P_3V_2}{P_1V_1}}\quad .\ .\ .\ .\ .\ (47)$$

If $V_1 = \infty$ and a and $n = 1$

$$= \frac{1 - \frac{V_1}{V_2}}{1 - \frac{P_3 V_2}{P_1 V_1}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (48)$$

Heat for Producing the Steam.—The total heat consumed per high-pressure cylinder full of steam in dynamical value, or ft.-lb. units, is, see equation (2),

$$H = V_1 D_1 [H_1 + J(t_f - 32^\circ)] \qquad (49)$$

and the efficiency is given by (3).

Maximum Efficiency.—In the compound engine now considered, if the two cylinders are of nearly equal size, one does almost no work, with slight expansion, and with a corresponding low efficiency. On the other hand, if there be a very great disparity of cylinder sizes, the low-pressure cylinder may exhaust the steam so rapidly from the receiver as to carry its pressure as low or even lower than the back pressure. At the limit of equal back and receiver pressure, no work will be done by the low-pressure cylinder, and the high-pressure cylinder exhausts, in effect, into the back pressure direct. Here again we have no working expansion, and a correspondingly low efficiency. It is evident that between these limits there exists a relation of sizes which will give a maximum of efficiency.

To determine this relation, assume a fixed volume of V_1 and of V', while V_2 is made to vary. In this way the quantity of steam used per stroke is invariable, so that the denominator of (3) is constant.

Hence, for the maximum of (3), we have only to examine the numerator, or to examine (37), (38), or (29), etc., according to contemplated accuracy. This could be done by working out several values from which to construct a curve, the maximum ordinate of which corresponds to the maximum sought. This plan is probably advisable for (37) and (38).

If we place the differential co-efficient of U'' with respect to V_2 from (39) equal zero, we get after reduction, for the case $V' = \infty$,

$$\frac{P_3}{P_1}=\left(\frac{V_1}{V_2}\right)^n\cdot\left(n\left(\frac{V_1}{V_2}\right)-(n-1)\right) \quad . \quad (50)$$

an equation which is irresolvable for the desired quantity, viz., $\frac{V_1}{V_2}$. But for a series of assumed values of $\frac{V_1}{V_2}$ the corresponding values of $\frac{P_3}{P_1}$ may be computed and tabulated. A sufficiently extended table would answer all cases, requiring conditions for the maximum efficiency.

If $n=1$, then (50) reduces to

$$\frac{P_3}{P_1}=\left(\frac{V_1}{V_2}\right)^2 \quad . \quad . \quad . \quad (51)$$

Eliminating $\frac{P_3}{P_1}$ between (50) and (39) we find the maximum of U'', or for the infinite receiver, with n as in (39);

$$U''_{\text{max.}}=P_1V_1\left\{1-\left(\frac{V_1}{V_2}\right)^n\left(1+n-n\frac{V_2}{V_1}\right)\right\} \quad . \quad . \quad . \quad . \quad . \quad (52)$$

From this, for $n=1$, or for (51) in (40) we obtain for $V'=\infty$ and $n=1$ the maximum of U'''

$$U'''_{\text{max.}} = 2P_1V_1\left(1 - \frac{V_1}{V_2}\right)$$

$$= 2P_1V_1\left(1 - \left(\frac{P_1}{P_3}\right)^{\frac{1}{2}}\right) \quad . \quad . \quad (53)$$

Conditions for Equal Work of Cylinders.—In practice it is probably desirable that the cylinders do equal work, particularly so in the Worthington pumping engine, where the especially important point is made of destroying the "water hammer" in connection with the water valve action, and of maintaining perfect uniformity of pressure and motion of the traveling water column.

To obtain an equation which shall express the conditions necessary for such equality of work of the two cylinders, we have only to place the numerator and denominator of (45) equal to each other, which becomes

$$\frac{P_3V_2}{P_1V_1} = \left(M + N\frac{V_2}{V_1}\right)\left(\frac{V_1}{V_2}\right)^n\left(1 + \frac{V_1}{V_2 + V'}\right)^{a-n} - 1$$

$$. \quad . \quad . \quad . \quad (54)$$

In using this equation it will be necessary to assume a set of volumes and find the ratio $\frac{P_3}{P_1}$, a satisfactory value for which may require several trials.

If $V' = \infty$ (54) reduces to

$$\frac{P_3V_2}{P_1V_1} = \left(1 + \frac{V_2}{V_1}\right)\left(\frac{V_1}{V_2}\right)^n - 1 \quad . \; . \; (55)$$

If $n = 1$, also,

$$\frac{P_3}{P_1} = \left(\frac{V_1}{V_2}\right)^2 \quad . \quad . \quad . \quad (56)$$

This last is the same as (51), which gives the condition for a maximum efficiency. Hence, for the case that the receiver has an infinite volume, and $n = 1$, the engine works with its maximum efficiency when the cylinders do equal portions of the work. When the receiver is eight or ten times as large as the low-pressure cylinder, this engine, with its valves, working as stated, is not far from working with its maximum efficiency.

VI. SOLUTION FOR THE SECOND VALVE MOVEMENT OF FIG. 21.

The complete diagram of steam action in continuity for this case is shown in

Fig. 23. The notation is as before, AC being the volume of the three parts V_1, V_2, and V'.

The piston motions are the same here as shown in Fig. 21, and, indeed, the same as in V. The sole cause for the difference in diagrams is in the valve movement, as indicated in the second part of Fig. 21.

The Complete Diagram and Indicator Cards.—When the H-piston has completed its forward stroke, all under the full boiler pressure P_1, the boiler steam is to be immediately cut off, and to be exhausted into the receiver. The resulting pressure is $BJ=DI=P$. The valve remains open, and ths H-piston stands, while the L-piston makes its stroke. The latter starts while the volume of active steam is that in the receiver and H-cylinder together, and equals $AD=V_1+V'$. Hence, the expansion line for this stroke is IH, drawn with the point A for the zero of volumes. Immediately on completing the stroke, the admission valve from the receiver is closed, retaining the

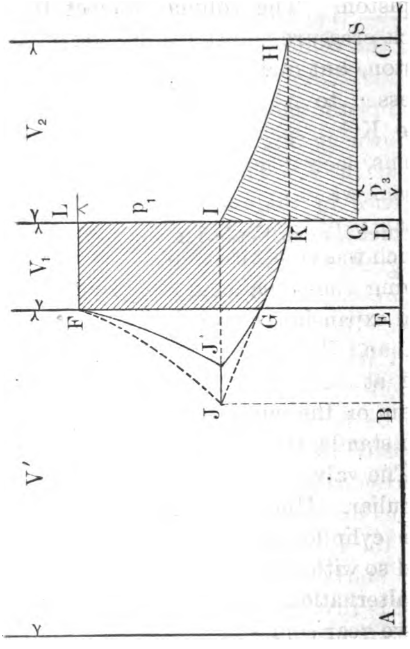

FIG. 23.

steam in the L-cylinder while its piston stands. Consequently the H-piston starts its stroke with a back pressure DK=CH of the terminal stroke of the

L-piston. The volume subject to this back pressure is AD at the start of the H piston, but during this stroke it is compressed to AE, giving the compression line KG. Then the valve immediately opens, as explained, to release the high steam which made this stroke. This steam expands against that in the receiver, which was compressed on the back stroke, giving the actual compression curve GJ′ and expansion curve FJ′; the pressures EG and EF meeting in a common pressure at J′. Then the L-cylinder starts again on the curve IH, while the H-piston stands, and so on in repetition.

The valve movement is thus seen to be peculiar. Here, instead of the valves of one cylinder all moving simultaneously, and so with the other cylinder, they move in alternation. This will complicate the valve-gear somewhat.

It is to be observed that by this arrangement, the high steam is at once released from the H-cylinder on completion of stroke, so that it is available to the L-cylinder. Also, in the L-cylinder, the steam is retained while that piston stands, instead of being at once exhausted; the object being to keep this cylinder hot as possible.

Nature of the Expansion and Compression Lines.—The point of zero of volumes for the curves HI and KGJ′ or J, is A, Fig. 23, while for FJ, or FJ′, it is D The back pressure line of the L-cylinder is QS.

Area of Diagrams and General Equations.—By aid of (7) and Fig. 23 we are able to write out

$$\text{Area GEDK} = P_k \frac{V_1 + V'}{a-1} \left\{ \left(1 + \frac{V_1}{V'}\right)^{a-1} - 1 \right\}$$
$$= P_k V_1 Q \quad . \quad . \quad . \quad (57)$$

if

$$Q = \frac{V_1 + V'}{V_1(a-1)} \left\{ \left(1 + \frac{V_1}{V'} \right)^{a-1} - 1 \right\} \quad . \quad (58)$$

and

$$\text{IDCH} = P_k \frac{V_1 + V_2 + V'}{a-1} \left\{ \left(1 + \frac{V_2}{V_1 + V'}\right)^{a-1} - 1 \right\}$$
$$= P_k V_2 S \quad . \quad . \quad . \quad (59)$$

if

$$S = \frac{V_1 + V_2 + V'}{V_2(a-1)} \left\{ \left(1 + \frac{V_2}{V_1 + V'}\right)^{a-1} - 1 \right\} (60)$$

For the particular case that $a = 1$, we have

$$(\mathrm{GEDK})_{a=1} = \mathrm{P}_k(\mathrm{V}_1 + \mathrm{V}')\,\text{hyp.log.}\left(1 + \frac{\mathrm{V}_1}{\mathrm{V}'}\right) \quad . \quad . \quad . \quad (61)$$

and

$$(\mathrm{IDCH})_{a=1} = \mathrm{P}_k(\mathrm{V}_1 + \mathrm{V}_2 + \mathrm{V}')\,\text{hyp.log.}\left(1 + \frac{\mathrm{V}_2}{\mathrm{V}_1 + \mathrm{V}'}\right) \quad (62)$$

in all of which P_k = the pressure DK = CH.

Now the work done in the high-pressure cylinder will be

$$\mathrm{FGKL} = \mathrm{P}_1\mathrm{V}_1 - \mathrm{GEDK} \quad . \quad . \quad (63)$$

and in the low-pressure cylinder it will be

$$\mathrm{IQSH} = \mathrm{IDCH} - \mathrm{P}_3\mathrm{V}_2 \quad . \quad . \quad (64)$$

For the engine working in continuity the work developed during a half cycle, or for one stroke of each cylinder, or by each high-pressure cylinder full of steam, will be the sum of (63) and (64), or

$$\mathrm{U} = (\mathrm{P}_1\mathrm{V}_1 - \mathrm{GEDK}) + (\mathrm{IDCH} - \mathrm{P}_3\mathrm{V}_2)$$
$$= \mathrm{P}_1\mathrm{V}_1 - \mathrm{P}_k\mathrm{V}_1\mathrm{Q} - \mathrm{P}_3\mathrm{V}_2 + \mathrm{P}_k\mathrm{V}_2\mathrm{S} \quad . \quad . \quad (65)$$

But in these equations P_k is as yet unknown. Regarding J and J′ as coincident on the adiabatic KG produced, then from the proportionality of horizontal intercepts between adiabatics, we may write

$$\frac{HK}{IJ}=\frac{AC}{AD}=\frac{V_2+V_1+V'}{V_1+V'}=1+\frac{V_2}{V_1+V'}$$

$$=\frac{V_2}{IJ}=\left(\frac{P}{P_k}\right)^{\frac{1}{a}}=1+\frac{V_2}{V_1+V'}.$$

Also,

$$\frac{P_i}{P_1}=\left(\frac{V_1}{IJ}\right)^n=\left(\frac{V_1}{V_2}\right)^n\left(1+\frac{V_2}{V_1+V'}\right)^n. \quad (66)$$

Hence

$$\frac{P_i P_k}{P_1 P_i}=\left(\frac{V_1}{V_2}\right)^n\left(1+\frac{V_2}{V_1+V'}\right)^{n-a}=\frac{P_k}{P_1}=\frac{P_2}{P_1} \quad (67)$$

This introduced in (65) gives in known terms the amount of work per high-pressure cylinder full of steam, viz.:

$$U=P_1V_1\left\{1+\left(\frac{V_2}{V_1}S-Q\right)\left(\frac{V_1}{V_2}\right)^n\left(1+\frac{V_2}{V_1+V'}\right)^{n-a}-\frac{P_3V_2}{P_1V_1}\right\} \quad . \quad . \quad . \quad . \quad (68)$$

If $a=n$, the term $\left(1+\frac{V_2}{V_1+V'}\right)$ drops out. For a and $n=1$,

$$U'=P_1V_1\left\{\begin{array}{l}1-\dfrac{V_1+V'}{V_2}\,\text{hyp.log.}\left(1+\dfrac{V_1}{V'}\right)\\ -\dfrac{P_3V_2}{P_1V_1}+\left(1+\dfrac{V_1+V'}{V_2}\right)\times\\ \qquad\text{hyp. log.}\left(1+\dfrac{V_2}{V_1+V'}\right)\end{array}\right\} \quad . \; . \; . \; . \; . \; . \; (69)$$

If $V'=\infty$, and a and n as in (68), we get

$$U''=P_1V_1\left\{1-\left(\frac{V_1}{V_2}\right)^n+\left(\frac{V_1}{V_2}\right)^{n-1}-\frac{P_3V_2}{P_1V_1}\right\} \quad . \; . \; . \; . \; . \; (70) \text{ or } (39)$$

the same as (39).

If $V'=\infty$, and a and $n=1$, we get

$$U'''=P_1V_1\left(2-\frac{V_1}{V_2}-\frac{P_3V_2}{P_1V_1}\right) \quad . \; (71) \text{ or } (40)$$

the same as (40).

One fact to be observed particularly, in comparing these equations with those for the V solution, is the perfect agreement of the expressions for the work U as soon as V' is made infinite. This is evidently as it should be, since for an infinite volume of receiver it would not matter whether the high pressure exhausted into it immediately on completion of stroke, or whether the exhaust were stayed each time during the half stroke; because the pressure could not vary appreciably in the infinite receiver during this time. For the infinite receiver it is only essential that equal weights be received from the high cylinder, and delivered to the low cylinder per stroke, the strokes being regarded as the same per minute for one as for the other cylinder.

The coincidence of the expressions for work, for the case of an infinite receiver, is therefore expected; and the fact of coincidence corroborates the analysis.

These equations (68) to (71) serve in calculations for duty where the whole work done for a period is compared with the coal consumed. In selecting the equation for use, judgment must be exercised as to the degree of approximation necessary in the case, and as to the proper values of *a* and *n*. Wet steam will require different values than dry, the value of *a* for such case being more properly

m, as found in the table following equation (1).

Pressure at Different Points of Steam Action.—To determine the pressure of the steam at different points of action in the engine, we observe first that for $V' = \infty$, the pressures at H, I, J, and K, Fig. 23, become one and the same; that is to say, the pressure in the receiver remains constant, and the indicator diagrams for the two cylinders are simple rectangles.

For the pressure at I or J see Eq. (66).

For the pressure at H or K see Eq. (67), where $P_k = P_2$.

These give the initial and terminal back pressures in the low cylinder, and the initial back pressure in the high cylinder. The terminal back pressure in the high cylinder is $P_g = EG$, for which, by aid of Fig. 23, we may write

$$\frac{P_g}{P_k} = \left(\frac{V_1 + V'}{V'}\right)^a = \left(1 + \frac{V_1}{V'}\right)^a.$$

Combining with (67),

$$\frac{P_g}{P_k} \cdot \frac{P_k}{P_1} = \left(1 + \frac{V_1}{V'}\right)^a \left(\frac{V_1}{V_2}\right)^n \left(1 + \frac{V_2}{V_1 + V'}\right)^{n-a} = \frac{P_g}{P_1} \quad \text{. . . . . . (72)}$$

If $V'=\infty$, these equations, as well as (66) and (67), reduce to

$$\frac{P_g \text{ or } P_i \text{ or } P_2}{P_1}=\left(\frac{V_1}{V_2}\right)^n \quad . \quad . \quad (73)$$

a common pressure, or a constant pressure of receiver, as above stated, for the case the receiver is infinite in volume. This equation also shows that n is the only exponent upon which the pressure of the infinite receiver depends. It also gives the pressure to which that of the receiver approaches as V' is made relatively very large.

Equation (67) shows that if $a=n$, as it will nearly for wet steam, the pressure at H or K, $=P_2$, will be entirely independent of V' whether $V'=\infty$ or not. In this case the larger the receiver the greater will be the proportion of work done by the high cylinder, and the less by the low cylinder, as indicated by Fig. 23. The pressure P_2 is here the pivotal pressure.

It is a curious fact that for $a=n$, the so-called pivotal pressures, for both valve movements shown in Fig. 21 have one and the same value; as indicated by equations (36) and (67) for $a=n$. That is to say, for given initial pressures and ratio of cylinder volumes the pressures of P_g of Fig. 22, and P_k of Fig. 23, are equal and constant, whatever the value of V'. Again it is immaterial to this pressure whether $a=n$, or $V=\infty$; and it is given by (44) or (73), another evidence that for the infinite receiver the mode of operation of valves is unimportant, whether according to 1st or 2d of Fig. 21.

The ratio of equations thus:

$$\frac{(42)}{(67)} > 1$$

shows that the second valve movement results in a greater ratio of expansion than the first when V' is not infinite.

Relative areas of Indicator Cards.—To find the relation between the amounts of work developed by the high pressure cylinder and the low, we may take their ratio thus:

$$\frac{\text{Wk.H.Cyl.}}{\text{Wk.L.Cyl.}}=\frac{1-Q\left(\frac{V_1}{V_2}\right)^n\left(1+\frac{V_2}{V_1+V'}\right)^{n-a}}{S\frac{V_2}{V_1}\left(\frac{V_1}{V_2}\right)^n\left(1+\frac{V_2}{V_1+V'}\right)^{n-a}-\frac{P_3V_2}{P_1V_1}} \quad \ldots\ldots (74)$$

For a and $n=1$;

$$=\frac{1-\frac{V_1+V'}{V_2}\text{hyp.log.}\left(1+\frac{V_1}{V'}\right)}{\left(1+\frac{V_1+V'}{V_2}\right)\text{hyp.log.}\left(1+\frac{V_2}{V_1+V'}\right)-\frac{V_3P_2}{V_1P_1}} \quad \ldots\ldots (75)$$

If $V'=\infty$, and a and n as in (74),

$$=\frac{1-\left(\frac{V_1}{V_2}\right)^n}{\left(\frac{V_1}{V_2}\right)^{n-1}-\frac{P_3V_2}{P_1V_1}} \quad \ldots\ldots (76)$$

If $V'=\infty$, and a and $n=1$,

$$=\frac{1-\frac{V_1}{V_2}}{1-\frac{P_3V_2}{P_1V_1}} \quad \ldots\ldots (77)$$

Equations (76) and (77) are seen to be identical with (47) and (48), which is evidently correct for the infinite receiver.

The heat required for making the

steam, also the expression for the efficiency is evidently the same here as in the first form of engine.

Maximum Efficiency.—The maximum of (68) or (69) may be found by the same process as indicated for (37) or (38). The maximum of (70) is the same as for (39), because the equations are identical. Hence for U″ the conditions for a maximum are obtained from (50); also (51) follows for $n=1$. Hence the maximum values of U″ and U‴ for the present case are found in equations (52) and (53).

But it is a remarkable fact that the maximum efficiency of this engine is exactly equal to that of the Woolf engine, without receiver, shown in Fig. 17 above. That is to say, when $a=n=m$, equation (15) is the maximum of (68). This can be proved by using for the expression of the work, the same as (68) obtained by aid of (57), and the corresponding one for the L-cylinder; also (67); and finally the relations

$$\frac{V_2}{V_1}=\frac{V_1+V'}{V'}=\frac{V_1+V_2+V'}{V_1+V'},$$

obtained from Fig. 23, on the supposition that V′ is made so small that the compression line KG is so raised that it extends from K direct to F. This compression line then becomes a cushion line, such that on the return stroke of the H-piston, the remaining steam is compressed and forced into the receiver until at the end of the return stroke the boiler pressure is just restored at F.

The size of the receiver for this special case is $V'=\frac{V_1^2}{V_2-V_1}$; also the isodynamic expansion now vanishes so that this maximum is general.

Conditions for Equal Work of Cylinders.—That the cylinders do equal work, the numerator and denominator of (74) must equal each other which condition gives

$$\frac{P_3V_2}{P_1V_1}=\left(Q+S\frac{V_2}{V_1}\right)\left(\frac{V_1}{V_2}\right)^n\left(1+\frac{V_2}{V_1+V'}\right)^{n-a}-1 \qquad . \quad . \quad . \quad . \quad (78)$$

the solution of which may be proceeded with as suggested for (54).

If $V' = \infty$, this expression reduces to the same as (55), as it should; also to (56), for $n = 1$. This last is seen to be the same condition as for the maximum efficiency, for $V' = \infty$ and $n = 1$, as stated for the first engine, in which respect the engines agree.

The Infinitude of Possible Pump Movements.—In all the preceding cases the piston motion has been regarded as such that when one piston moves for making its complete stroke the other piston stands still at the end of its cylinder, and *vice versa*, as first done in the celebrated Worthington pumping engine, and recently in many others. But compound pumping engines are in use in which we find the three parts V_1, V_2 and V' as above, but with different piston and valve motions, as, for instance, in case of certain Cope & Maxwell pumping engines.

Respecting the possible variety of engines due to unlimited suppositions for piston motion, or valve motion, or both, it appears to be infinite. For almost any one of these the diagrams corresponding with Figs. 22 or 24 become exceedingly difficult to delineate, except for the simple case $V' = \infty$. For instance, there might be the tarrying of the pistons, each for half or other fraction of its time; but with the relative period of strokes indifferent. The steam cylinders might be of equal volume, while one makes twice as many strokes as the other, and thus obtain expansion. This is admissible with or without tarrying of pistons. While the high piston tarries, its valve might open into the receiver at any point of time in the period of tarrying. And the low cylinder might close its valve from the receiver at any point in the tarrying of its piston. A comparatively simple case is that where the pistons do not tarry, and where the valves all work promptly on the termination of the strokes of their respective pistons. But here the periods of strokes of pistons might be in any given relation.

VII.—STROKES CO-INITIAL AND CO-TERMINAL; WITHOUT TARRYING.

For this case a little consideration will show that it matters not at which end of stroke one piston is, while the other starts at a particular end; because, for

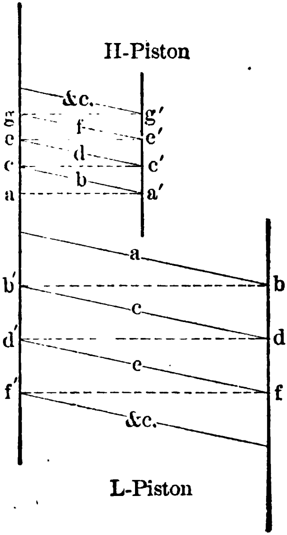

FIG. 24.

either one piston, the same changes occur at one end as at the other end of stroke. And let there be no tarrying, but both pistons moving continuously. For this, a little consideration will show that this engine may be rotative, with cranks at 180°, and one cylinder to each, and that it is the same as any, Cases I to IV, and Figs. 11 to 14.

VIII.—STROKES INTERFLUENT, WITHOUT TARRYING.

As an example of a case not quite so simple, and to show the effect of interruption due to the exhaust into the receiver from the high cylinder while the low cylinder is making its stroke, let the reversal of stroke for either piston be at the midstroke of the other. Then Fig. 24 may serve to indicate the relation between piston positions for continuity. The part H refers to the high cylinder, and L to the low. Now suppose H on the point of exhausting into the receiver, and let *a* represent the beginning and *a′* the end of this operation. This takes place at the midstroke, *a*, of the L-piston, as shown in the L part of the diagram. As H exhausts, the volume of receiver and connections will increase in the operation by an amount *aa'* on a proper scale. Now the H-piston travels from *a'* to *b*, while the L-piston travels from *a* to *b*. Then the L-cylinder exhausts into the air or condenser, and its volume is cut off from the receiver to begin a new cylinder full. This volume cut off from the receiver is *bb'*, as shown on the L part of the figure. This takes place at the midstroke *b* of the H-piston; see Fig. 24. And thus these operations continue, as can readily be traced from Fig. 24.

Now when a piston makes a half-stroke, the change of volume of the receiver and connections will not change by that amount alone, because two pistons are in action simultaneously, and the change of volume just referred to will be due to the combined movements of pistons, one of which (the H) is compressing, and the other (the L) is expanding this volume. We observe that the high piston *always* compresses, and the low expands this volume.

The letters *a*, *b*, *c*, *d*, etc, on Fig. 24, for any one letter denotes a single point of time, so that by referring to any single letter we can at once see the relative positions of both pistons. The stretches between *a* and *a′*, *c* and *c′*, etc., or *b* and *b′*, *d* and *d′*, etc., indicate shifting of action from end to end. Thus, when the exhaust is completed from one side of the H-piston, it immediately begins from the other side.

For the simultaneous positions, *a*, we see that the L-piston is at midstroke, while the H-piston is exhausting. Just before beginning the exhaust, *a*, the total volume in the receiver and connections is $V' + \frac{V_2}{2}$, and just after it is $V' + \frac{V_2}{2} + V_1$, as can easily be traced. For the point *b*, the H-piston is at midstroke, and the L-cylinder is on the point of exhausting and changing the volume from

$$V' + V_2 + \frac{V_1}{2} \text{ to } V' + \frac{V_1}{2}, \text{ etc., etc.}$$

These changes of volume of receiver and connections are better shown in Fig. 25.

The Complete Diagram.—The shaded areas of Fig. 25 are the indicator cards for the cylinders. The volumes indicated show to which either card belongs. All besides the shaded cards are construction lines used in obtaining the expansion lines and cards.

The lower part of the figure shows the variations in volume. $AB = V'$ is the volume of the receiver itself. At a we have the L-piston at midstroke, while the volume in the receiver and connections is $V' + \frac{V_2}{2}$ as shown. Also the H-cylinder is ready to exhaust, and add the volume $aa' = V_1$. The line at $a'b$ is the variation of volume in the receiver and connections for the half-strokes shown for $a'b$ and ab in Fig. 24. Both pistons move for this, one to reduce, and the other to enlarge the volume considered. The result is an enlargement to the point

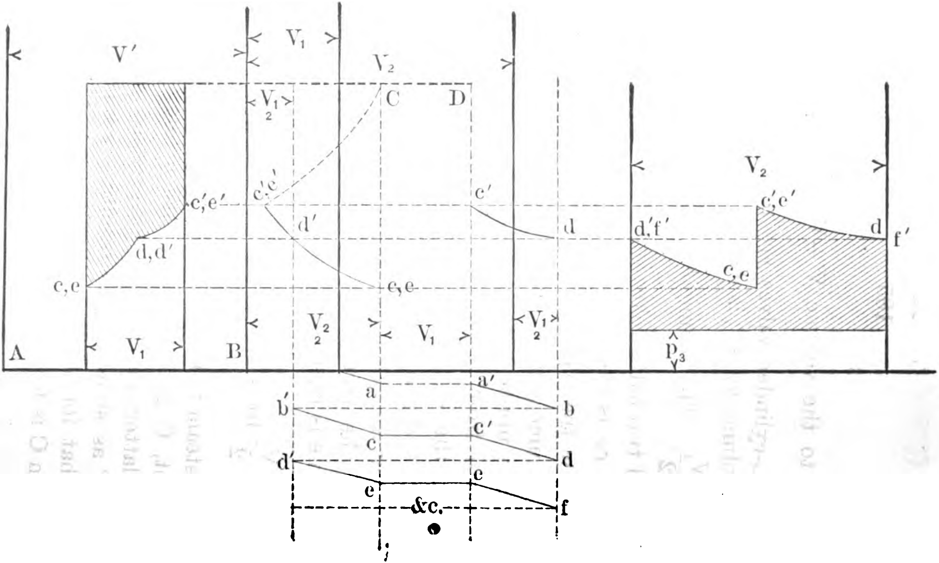
V′
V₁
V₂
$\frac{V_1}{2}$
C
D
c′,e′
c′,e′
d,d′
c,e
d′
e,e
c′
d
A
V₁
B
$\frac{V_2}{2}$
V₁
$\frac{V_1}{2}$
V₂
c′,e′
d′,f′
d
f′
c,e
p₃
a
a′
b′
b
c
c′
d′
d
e
e
f
&c.

Fig. 25.

b, or to the volume $V'+V_2+\frac{V_1}{2}$. Then the L-cylinder valves are reversed, and the volume $bb'=V_2$ is cut off, leaving only $V'+\frac{V_1}{2}$. The volume $b'c$ is due to combined two half-strokes similarly as in $a'b$. Then cc' is like aa', etc.

Now the expansion line for the change of volume $a'b$, or $c'd$, etc., is $c'd$ above, in the upper part of Fig. 25. This is drawn from A as the zero of volumes. Also the expansion line for $b'c$, or $d'e$, etc., is $d'e$ above. For this the zero is also at A. When the H-cylinder exhausts into the receiver and connections, the volume is raised from

$$V'+\frac{V_2}{2} \text{ to } V'+\frac{V_2}{2}+V_1, \text{ or from C to D;}$$

the steam following from a high-pressure point, C, giving the expansion line Cc', the latter meeting the compression line $cd'c'$ as shown. The resulting pressure is that for the point c'. The expansion from C is for a zero of volumes at D, as will be understood after studying Figs. 22 and 23.

The diagrams are drawn at one side and the other, to avoid confusing the figure.

Now as the H-cylinder exhausts, the pressure becomes that at *c′*, when the H-piston begins its back stroke with a like back pressure. Owing to an increase of volume during this back stroke, the line that would be a compression line for this cylinder alone, becomes a falling or expansion line as shown; first, as far as to *d* while the L-piston makes a half-stroke; and then as continued to *c* for the other half, but with a diminishing volume in the receiver and connections. Hence, the lower line *c′de* of the H-cylinder card is a broken line as shown.

The L-cylinder expansion line is in two parts also, that at *d′e* being due to the fall of pressure for the first half of stroke as traceable by lettering. At the mid-stroke there is a sudden rise of pressure *cc′*, due to the exhaust of the H-cylinder into the receiver. From this point expansion continues on a new line due to a different pressure and volume.

Without entering into analysis, it is probably safe to predict that for an infinite volume the pressure in the receiver will be the same as by (14) and other equations; that is to say, the efficiency is still the same as previously given for an infinite receiver, and nearly so for receivers relatively large. Hence, when the receiver is very large as compared with the cylinders, and when there is no cut-off to either cylinder, the efficiency of engine remains very nearly the same, irrespective of the piston motion or valve motion. Whether there be an advantage in any form of valve or piston motion, for a finite receiver over the infinite, will perhaps be best shown by the numerical results subsequently given.

For the large receiver it is evident that expansion may be obtained equally well, either with equal strokes and unequal cylinders, or unequal strokes with equal cylinders, or both combined.

IX.—THE TANDEM DUPLEX COMPOUND PUMPING ENGINE.

This engine differs essentially from the preceding by having four cylinders and no receiver.

Such an engine is shown in Fig. 9, in which there are four steam cylinders, two of one size and two of another size; one large and one small cylinder are seen arranged in line of each one of the two piston-rods. The two on one piston-rod form one complete engine of the Woolf form.

Thermodynamically, this engine might be treated without the consideration of duplex, because in reality there are two distinct engines in this form of duplex. We will treat one part as covering the whole in principle.

Here one cylinder may be placed in line with the other, on a common piston-rod, and when so related the arrangement is called "tandem." But the arrangement is immaterial, provided the strokes are co-terminous and without tarrying. The steam is to be delivered directly from the high-pressure cylinder into the low, without the intervention of the receiver. Also, here the steam is supposed to be admitted to the H-cylinder for the full stroke, and full pressure of boiler; it is then to be exhausted, or transferred to the other cylinder during the whole stroke, which stroke is in common for the two pistons. In this way the volume of steam which occupies the H-cylinder at the end of one stroke, is expanded to the volume of the L-cylinder by the end of the next stroke.

Equation (15) is to be employed for this engine if there be no receiver; or, in the existence of a receiver, even including the intermediate communicating pipe as such, equation (14) will apply.

In these discussions the engine itself is the only part of the machine brought into account, but in the practice of steam pump engineering there are essential considerations relative to the pump as well, some of which have already been referred to.

NUMERICAL RESULTS FOR COMPARISON.

A few results have been computed and collected in the following table:

TABLE OF RESULTS FOR THE V, VI AND IX CASES.

P_1	$\frac{P_1}{P_3}$	$\frac{V_2}{V_1}$	*Effy.* V. $V'=2V_2$	*Effy.* VI. $V'=2V_2$	*Effy.* $V'=\infty$, and max. of V.	*Effy.* $V'=\infty$, $n=1$, for VI.	*Effy.* Tandem. and max. of IX.	$\frac{\text{Work in } V_1}{\text{Work in } V_2}$				$\frac{\text{Press. of admission, } P_1}{\text{Press. of exhaust, } P_2}$			
								V.	VI.	$V'=\infty$.	$V'=\infty$ $n=1$	V.	VI.	$V'=\infty$.	Tandem. IX.
1	2	3	4	5	6	7	8	9	10	11	12	13.	14	15	16
15.60	4.31	2	.0597	.0790	.0718	.0700	.0842	1.77	.78	1.01	1.	1.96	2.27	2.08	2.17
37.68	10.41	3	.0824	.1055	.0968	.0962	.1238	1.51	.86	1.03	1.	3.03	3.45	3 13	3.45
71.49	19.75	4	.0974	.1230	.1126	.1114	.1547	1.47	.83	1.04	1.	4.17	4.76	4.35	4.76
119.2	32.91	5	.1060	.1326	.1217	.1213	.1755	1.47	.90	1.05	1.	5.26	5.88	5.26	6.25

For the first column, (37) was used, and for the second (68); that is, the complete formulas were put to the test.

The feed-water was assumed at the temperature 149° F., and the back pressure 3.62 pounds per square inch, absolute.

The exponent a was taken at $\frac{3}{4} = 1.3333$, n at 1.0456, and for the tandem engine, m was taken at 1.135. The value of a is probably too high for ordinary practice where steam is likely to be supersaturated; probably the best value for practice lies between 1.1 and 1.2.

In looking over the table we at once notice the glaring fact that the tandem engine excels all the other pumping engines in efficiency, and by a considerable percentage. At 23 pounds per square inch apparent pressure, the excess of efficiency of the tandem over the VI is about 17 per cent. of the efficiency of the latter. For 104 pounds per square inch apparent pressure, the same is about 32 per cent.

A second important fact respecting the

two valve movements, viz., that of the V and VI is that the latter is a high percentage above the other.

A third important fact appearing in the table, is that in the VI movement, the efficiency is a very considerable percentage greater when the receiver is small than when it is infinite, a fact in support of the conclusions of solutions of VI and VII. Compare columns 5 and 6. For the 5, the receiver is only twice as large as the low pressure cylinder, and yet the efficiency here is from 8 to 10 per cent. higher than for the engine with an infinite volume.

For V, however, the case is the reverse, but in a greater ratio; that is to say, the efficiency is lowered from 13 to 17 per cent., by changing the receiver from an infinite volume to one only twice as large as the low-pressure cylinder.

As to the value of n, it appears that the efficiency does not change materially when n changes from 1 to 1.0456.

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